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SUPER-RESOLUTION RESTORATION OF CONTINUOUS IMAGE SEQUENCE USING THE LMS ALGORITHM

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ABSTRACT

In this paper, we propose computationally efficient **super-resolution restoration algorithm** for blurred, noisy and down-sampled continuous image sequence. The proposed approach is based on a Constrained Least Squares (CLS) super-resolution algorithm, applied recursively in time. Updating equation based on the instantaneous squared error gradient gives a Block-LMS version algorithm. Adaptive regularization term is shown to improve the restored image sequence quality by forcing smoothness, while preserving edges. The computation complexity of the obtained algorithm is of the order $O\{L^2 \log(L)\}$ per one output image, where L^2 is the number of pixels in the output image. Simulations carried out on test sequences prove this method to be applicable, efficient, and with very promising results. Frequency domain version (FDBLMS) of the algorithm is proposed for special cases, with further improvement in computational complexity, and rate of convergence.

1. INTRODUCTION

This paper addresses super-resolution restoration of continuous image sequence assumed blurred, noisy and down-sampled. The general purpose is restoring the source image sequence with improved (super-resolution) resolution.

Restoration of image sequences was not as extensively treated in the literature as the single image restoration problem counterpart [1], mainly because of the computational complexity and memory requirements involved. The existing published results so far, regarding image sequences restoration, correspond to simplified variations of the presented restoration problem, which can be generally devided into three sub-groups: (i) Restoration of continuous image sequences with no treatment of the possibility of loss of resolution [2-4], (ii) Super-resolution algorithms with no treatment of the continuous flow of images in time [5-7], and (iii) Multiple restoration algorithms which generally assume no motion between the images, and do not treat the continuous flow of images in time [8,9].

In this paper, a novel approach is introduced for the task of restoration of a continuous image sequence. This approach starts by generalizing the existing super-resolution algorithms, and directs them toward the problem at hand [10].

Adopting a similar concept to the one proposed by [6,7], and using sparse matrices notation, a new and traceable model is suggested for the classic super-resolution restoration problem. (i.e. - restore one super-resolution image from several measured low-resolution blurred and noisy image). Applying the CLS results with linear sparse system of equations for the computation of the restored super-resolution image. Iterative relaxation algorithms are proposed for an efficient solution of this linear equations set.

Following the above framework, a new model for the generalized problem of restoration of continuous image sequence with super-resolution is proposed. Using this model and the constrained least squares (CLS) method, recursive equations in time are obtained for the computation of the estimated restored output image sequence. The presented concept operates as a classical super-resolution algorithm using infinite history data as a basis for the construction of each output image.

By updating the output image sequence using the instantaneous CLS error gradient, a Block-LMS version algorithm is obtained for the generalized restoration task. Applying this Block-LMS adaptive filter [11] on the proposed super-resolution problem results in a very efficient restoration algorithm. By assuming that the (space and time variant) blur function, the optical flow, the noise auto-correlation matrix and the resolution ratio are all arbitrary but known, the algorithm developed is capable of converging to the optimal restored high resolution image sequence.

The computation complexity of the obtained algorithm is of the order $O\{L^2 \log(L)\}$ per one output image, where L^2 is the number of pixels in the output image. Simulations carried out on test sequences prove this method to be applicable, efficient, and with very promising results.

Based on the BLMS algorithm, a frequency domain version (FDBLMS) [12] of the algorithm is proposed, for the case of linear space invariant blur and global translational motion in the scene. This FDBLMS algorithm enables a further reduction in the computational complexity and an improvement in the convergence rate.

The paper is organized as follows. In section 2 we present the new model and the resulting CLS restoration approach for the classical super-resolution restoration problem. Section 3 focuses on the generalization of the classical super-resolution approach, and the obtained BLMS restoration algorithm. Experimental results are shown in section 4, and conclusion is presented in section 5.

2. LEAST SQUARES SUPER-RESOLUTION

We start our presentation by treating the classical super-resolution restoration problem, where one super-resolution image is to be restored, using several measured low-resolution blurred and noisy images.

Given are N measured (lexicographicaly ordered) images $\{\underline{Y}_k\}_{k=1}^{N}$ of size $[M_k \times M_k]$. We assume that an ideal super-resolution image $\underline{X}$ exists of size $[L \times L]$, where $L \geq M_k$. The measured images are connected to the ideal image via the relationship:

$$\underline{Y}_k = D_k H_k F_k \underline{X} + \underline{E}_k \quad 1 \leq k \leq N \tag{1}$$

where D_k is a decimation operator, H_k is a Linear Space Variant (LSV) blur operator, and F_k is the geometric warp operator performed on the ideal image to generate the k-th measurement. $\underline{E}_k$ is an additive Gaussian noise, uncorrelated with the measurements and the ideal image, and with auto-correlation matrix W_k^{-1}. All the above matrices are assumed known. Discussion on this assumption is beyond the scope of this paper (see [10] for such a discussion). We define a scalar Least Square (LS) error by:

$$\varepsilon^2 = \sum_{k=1}^{N} \left\| \underline{Y}_k - D_k H_k F_k \underline{X} \right\|_{W_k}^2 \tag{2}$$

Minimization of this squared error with respect to the ideal unknown image by direct derivation yields the following equations:

$$\begin{aligned} \underline{P} &= \sum_{k=1}^{N} \left[D_k H_k F_k \right]^T W_k \underline{Y}_k \\ R &= \sum_{k=1}^{N} \left[D_k H_k F_k \right]^T W_k D_k H_k F_k \\ &\Rightarrow R \underline{\hat{X}} = \underline{P} \end{aligned} \tag{3}$$

Direct solution of the above linear system is impossible for typical sizes $(L >> 50)$. Iterative methods such as Steepest Descend, Conjugate Gradient, Successive Over-relaxation and Multi-Grid methods suggest themselves.

Note that there is no guarantee for single solution since the matrix R is a sum of semi-positive definite matrices, and thus is generally positive semi-definite. This means that iterative solution of the above linear set of equations will give a result which is dependent on the initialization [6]. These problems are overcome by adding the regularization term $\beta \left\| S \underline{X} \right\|_V^2$ to the squared error in equation (2), where S is smoothing operator, and V is diagonal positive definite matrix, weighing each pixel smoothness according to our a-priori knowledge [1]. This regularization term also changes the restoration solution from inverse filter to a Tichonov-Miller restoration, which is equivalent to the Wienner-like restoration [1].

The proposed approach is a direct CLS application to the super-resolution problem. By applying a version of the steepest descend iterative algorithm we get the Iterative Back projection method presented by Irani & Peleg [6]. By assuming global translational motion, space invariant blur, and homogeneous noise we can use 2-D DFT to transform equation (3) and get a frequency domain super-resolution method. The resulting equations-set can be easily shown to reduce to block-diagonal one [10], equivalent to the one presented in [5].

Each scalar term within the summation in equation (2) represents an ellipsoid constraining the required super-resolution ideal image, posed by the corresponding measured image. Since CLS is equivalent to the method of bounding ellipsoids [9], and the bounding ellipsoids method is closely related to the Projection Onto Convex Sets (POCS) method, our approach is similar to the one presented in [7]. Since our suggested restoration process is iterative in nature, projections onto constraints represented by convex-sets can be applied within each iteration to improve error and computational performance [1,7].

Summarizing this section, we have presented a CLS approach toward the classical problem of super-resolution restoration, and pointed out several iterative relaxation methods for the practical implementation of the restoration task. The presented approach enables treating different LSV blur, different non-homogenous additive noise, different arbitrary smooth motion, and different rational decimation factor for each measured image.

3. CONTINUOUS IMAGE RESTORATION

Let us now turn to define and treat a new problem which is a generalization of the classical super-resolution restoration problem.

Given is a measured image sequence $\{\underline{Y}(t)\}_{t\geq 0}$ of size $[M \times M]$. We assume that an ideal super-resolution image sequence $\{\underline{X}(t)\}_{t\geq 0}$ of size $[L \times L]$ each exists, where $L \geq M$. The ideal image is connected to all casual measured images $(0 \leq k \leq \infty)$ via the relationship:

$$\underline{Y}(t-k) = DH(t-k)F(t,k)\underline{X}(t) + \underline{E}(t,k) \qquad (4)$$

This equation is similar to equation (2), with indexes replaced by temporal correspondance. As before, D is a constant decimation operator, $H(t-k)$ is the Linear Space & Time Variant (LSTV) blur operator, and F(t,k) is the backward geometric warp operator performed on the ideal image $\underline{X}(t)$ to geometrically match the measured image $\underline{Y}(t-k)$. $\underline{E}(t,k)$ is an additive Gaussian noise, uncorrelated with all the casual measurements and the ideal casual image sequence, and with auto-correlation matrix $W^{-1}(t,k)$. All the above matrices are assumed known as before, with the same justification [10].

We define a scalar instantaneous Least Square error by:

$$\varepsilon^2(t) = \sum_{k=0}^{\infty} \left\| \underline{Y}(t-k) - DH(t-k)F(t,k)\underline{X}(t) \right\|^2_{W(t,k)} \qquad (5)$$

This squared error is similar to the error defined for the LS classical super-resolution problem, presented earlier. Minimization of this temporal squared error with respect to $\underline{X}(t)$ by direct derivation yields the following equations:

$$\begin{aligned} &\underline{P}(t) = \lambda F^T(t,1)\underline{P}(t-1) + H^T(t)D^T W_0 \underline{Y}(t) \\ &R = \lambda F^T(t,1)R(t-1)F(t,1) + H^T(t)D^T W_0 DH(t) \qquad (6) \\ &\Rightarrow \quad R(t)\hat{\underline{X}}(t) = \underline{P}(t) \end{aligned}$$

where we assume:

1. $W(t,k) = \lambda^k W_0$
2. $F(t,k) = F(t-k+1,1) \ldots\ldots F(t-1,1)F(t,1)$
3. $F(t,0) = I$

The obtained equations form a recursive process for the computation of $\hat{\underline{X}}(t)$. Although there is a resemblance to the theoretic Recursive Least Squares (RLS) algorithm, a simple recursive update to the restored image $\hat{\underline{X}}(t)$ is not possible [10]. As before, iterative methods can be applied per each time t, using all the improvements suggested before (regularization, POCS etc.).

Adopting the LMS algorithm idea [11], we can use the following update idea:

$$\hat{\underline{X}}(t) = G(t,1)\hat{\underline{X}}(t-1) - \frac{\mu}{2} \left. \frac{\partial \varepsilon^2(t)}{\partial \hat{\underline{X}}(t)} \right|_{G(t,1)\hat{\underline{X}}(t-1)} \qquad (7)$$

where G(t,1) is the forward geometric warp, given as the pseudo-inverse of the matrix F(t,1). The idea behind equation (7) is to update the previous result by warping it geometricaly forward (equivalet to performing motion compensation estimation), and adding an update term using the instantaneous squared error gradient, which advances according to the steapest decend rule.

Performing several trivial algebraic steps and assuming that the previous result are near optimal results in the following update equation:

$$\begin{aligned}\hat{\underline{X}}(t) &\cong G(t,1)\hat{\underline{X}}(t-1)+ \\ &+\mu H^T(t)D^T W_0\left[\underline{Y}(t)-DH(t)G(t,1)\hat{\underline{X}}(t-1)\right]\end{aligned} \quad (8)$$

This is an adaptive process to compute $\hat{\underline{X}}(t)$ using the motion compensated previous output image, and a vector update. Since the update is done per each image, the result is Block-LMS algorithm (simple LMS algorithm corresponds to an update per pixel).

Adding the regularization term $\beta\|S\underline{X}(t)\|^2_{V(t)}$ to the instantaneous squared error in equation (6), and performing all the above steps, the update equation becomes:

$$\begin{aligned}\hat{\underline{X}}(t) &\cong G(t,1)\hat{\underline{X}}(t-1)+ \\ &+\mu H^T(t)D^T W_0\left[\underline{Y}(t)-DH(t)G(t,1)\hat{\underline{X}}(t-1)\right]+ \\ &+\beta S^T V(t)SG(t,1)\hat{\underline{X}}(t-1)\end{aligned} \quad (9)$$

Convergence properties of the constrained and the unconstrained algorithms to the ideal sequence are disscussed in [10]. The choise of μ controls the speed of convergence, the excess steady-state error etc., with similar behaviour to the theoretic LMS algorithm [11,12].

Frequency domain method can be suggested by transforming equation (8). If the matrices G(t,1), H(t) and W_0 are block Toeplitz, and D represents space invariant sampling, multiplying by the 2-D DFT matrix T, we get a frequency domain update equation which is simpler to perform.

Such approach improves only the overall computational requirements [10], with no change in the convergence rate. By applying the 2-D DFT to equation (6), a block-diagonal linear system is propagated in time using very low computational burden, and very fast convergence rate.

4. SIMULATION RESULTS

The new BLMS continuous image restoration algorithm was applied to a synthetic image sequence containing 100 images, each of size $[25 \times 25]$, containing rotating graphics and text. This sequence was generated from ideal sequence of 100 images of size $[50 \times 50]$, using 3×3 uniform LSI blur and 2:1 decimation in each axis.

The parameters used in the restoration algorithm are: $\mu = 2$ (update step size), $\beta = 0.01$ (the smoothness constraint coefficient), and $W_0 = I$ (the model error weighting matrix). The motion flow was assumed known. The smoothness penalty was applied by using the Laplacian operator. The weighting matrix for the smoothness penalty was constructed by interpolating the current measured image, computing a squared Laplacian of it $L^2(i,j)$, and using the value:

$$V(i,j) = \frac{1}{1+\alpha L^2(i,j)} \quad (10)$$

as weighing the smoothness of the (i,j)-th output pixel ($\alpha = 9/\max\{L(i,j)\}$ so that $0.1 \le V(i,j) \le 1$). This way, smooth areas are forced to be smooth in the restored output, whereas non-smooth areas are not penalized for non-smoothness. This idea matches also the human psycho-physics vision properties (the masking effect), according to which the human viewer is not sensitive to noise in active regions of the image.

Figure 1 shows the 100-th ideal, measured, LMS output and smoothed LMS (with regularization) output images.

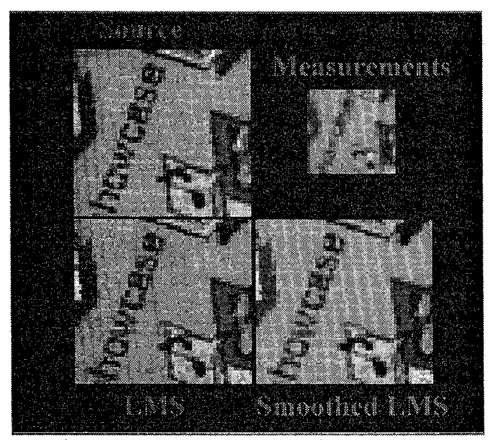
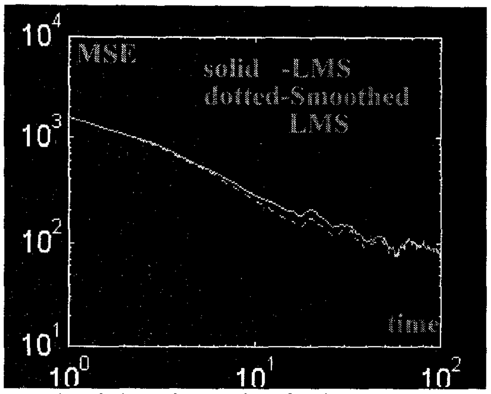


Figure 1 - the 100-th image in the source, measured, and restored (LMS & smoothed LMS) sequences

Graph 1 shows the mean square error as a function of time for the LMS and the smoothed LMS algorithms.



Graph 1 - MSE vs. time for the LMS and the smoothed LMS algorithms

As can be seen from the above figure and graph, the restoration quality achieved is very encouraging, both for the LMS and the smoothed LMS algorithms. The smoothed LMS algorithm in the presented case indeed supplies a slightly smoother result in silent background regions, with lower overall mean square error.

5. CONCLUSION

In this paper we have presented a new approach towards the classical problem of super-resolution restoration of single high-resolution image from several measurements. This approach which was based on a sparse matrices model and the CLS was shown to generalize the existing super-resolution methods.

Based on the above method, we have presented a generalized super-resolution problem where the purpose is the restoration of continuous image sequence with improved resolution. We have shown that the problem can be reduced to several recursive equations propagating in time. Based on these equations, a Block-LMS restoration algorithm was proposed. This approach was demonstrated with very encouraging results, both from the computational and the output quality aspects. A frequency domain version of the restoration algorithm for block Toeplitz matrices was also shown.

One direct consequence of the presented work is the need for simple, yet reliable, algorithms for the tasks of motion estimation for image sequences, and the estimation of the blur function. Both these components are required in order to present an autonomous process of continuous image sequence restoration with super-resolution. These issues are currently under investigation.

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