Another Take on Patch-Based Image Processing *

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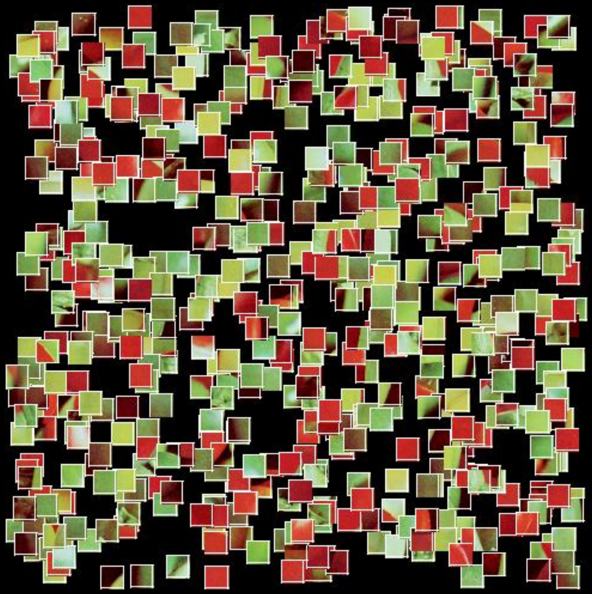
Patch-Based Processing of Images

In the past decade we see more and more researchers suggesting to process a signal or an image with a paradigm of the form:



Break the given image into overlapping (small) patches

Operate on the patches separately or by exploiting inter-relation between them

Put back the resulting patches into a result canvas

Patch-Based Processing of Images

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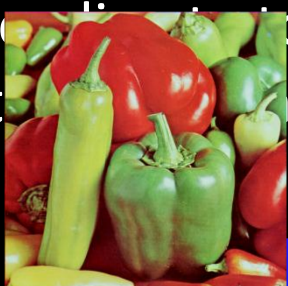
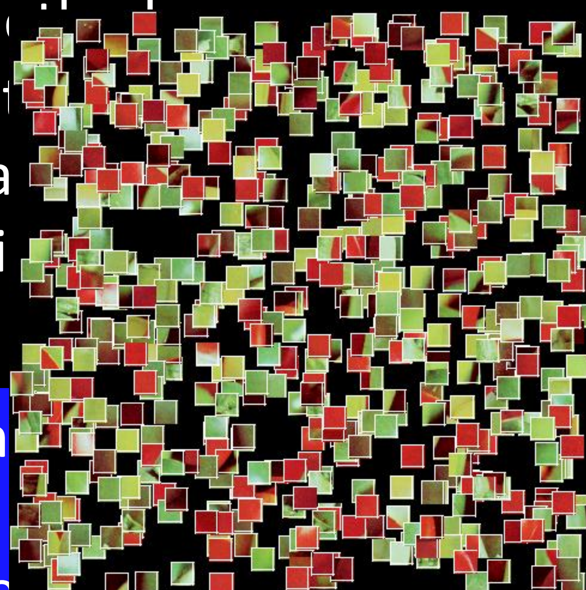




ommon th es are believed
to exhibit a etrical form in
the e reside in

Patches … Patches … Patches …

Who are the researchers promoting this line of work?
Many leading scientists from various places

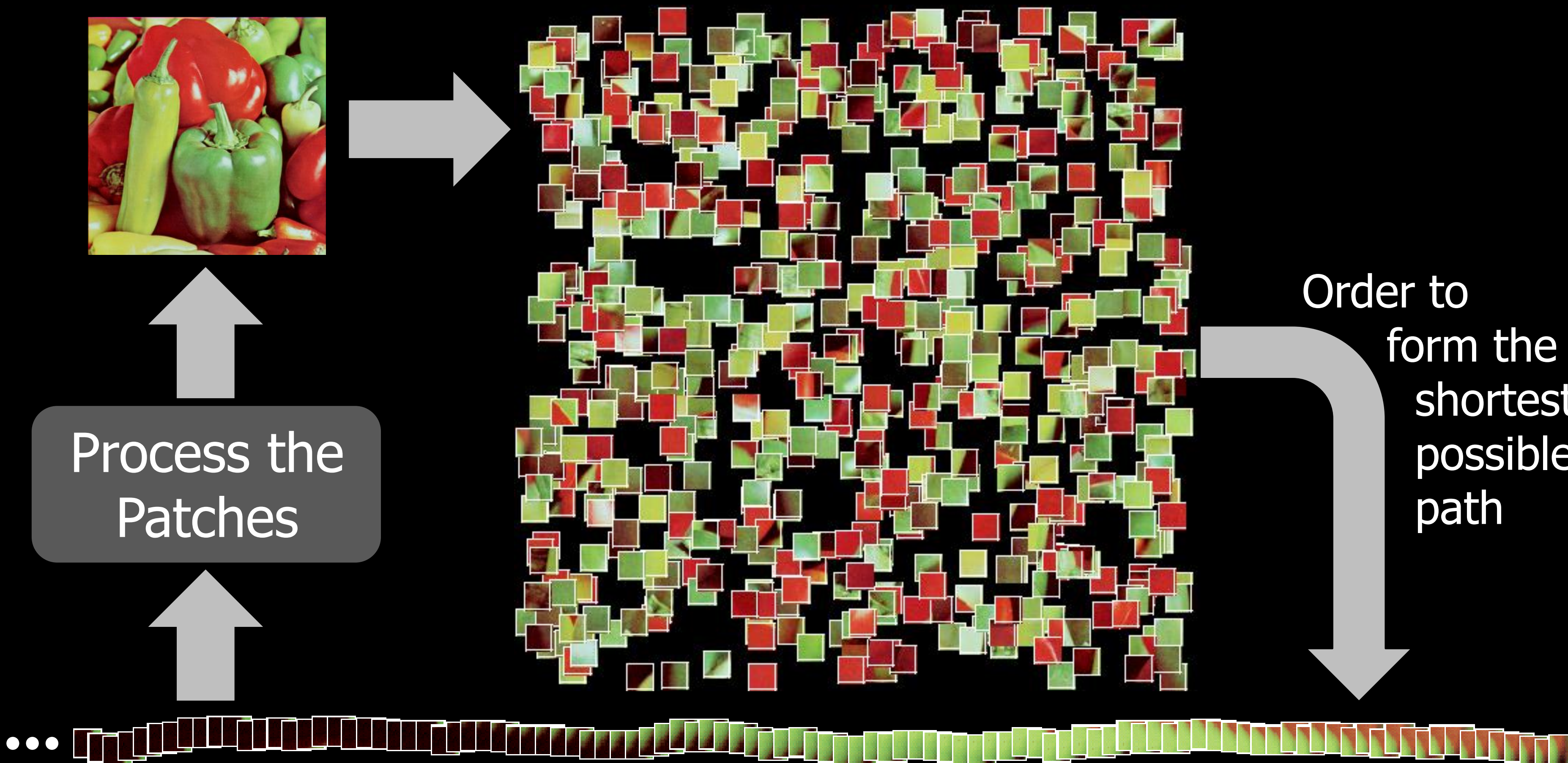
Sorry if
I forgot YOU

Various Ideas:

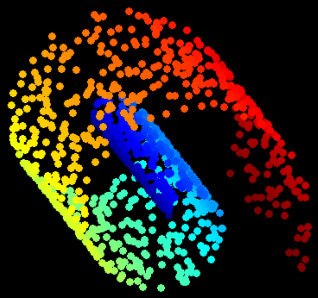
- Non-local-means
- Kernel regression
- Sparse representations
- Locally-learned dictionaries
- BM3D
- Structured sparsity
- Structural clustering
- Subspace clustering
- Gaussian-mixture-models
- Non-local sparse rep.
- Self-similarity
- Manifold learning
- …

This Talk is About …

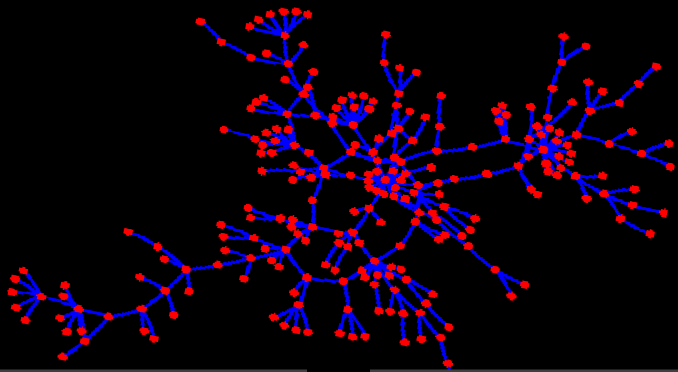
A different way to treat an image using its overlapped patches



Surprisingly, This Talk is Also About …



Processing of Non-Conventionally Structured Signals



Many signal-processing tools (filters, transforms, …) are tailored for uniformly sampled signals

Now we encounter different types of signals: e.g., point-clouds and graphs. Can we extend classical tools to these signals?

Our goal: Generalize the wavelet transform to handle this broad family of signals

In the process, we will find ourselves returning to “regular” signals, handling them differently

In fact, this is how this work started in the first place

Part I – GTBWT

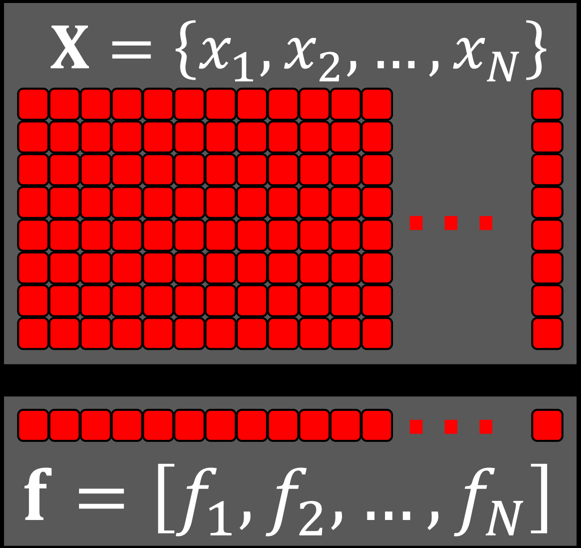
Generalized Tree-Based Wavelet Transform – The Basics

This part is taken from the following two papers:

- I. Ram, M. Elad, and I. Cohen, "Generalized Tree-Based Wavelet Transform", IEEE Trans. Signal Processing, vol. 59, no. 9, pp. 4199–4209, 2011.
- I. Ram, M. Elad, and I. Cohen, "Redundant Wavelets on Graphs and High Dimensional Data Clouds", IEEE Signal Processing Letters, Vol. 19, No. 5, pp. 291–294 , May 2012.

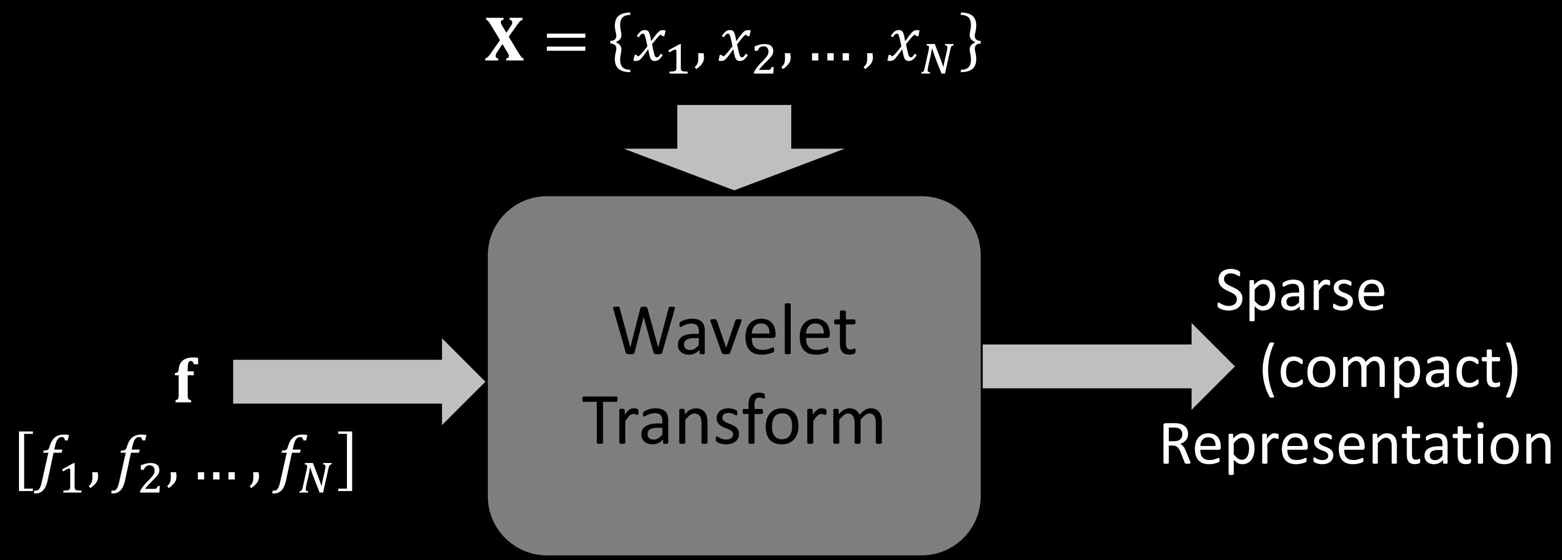
Problem Formulation

- We start with a set of points $\mathbf{X} = \{x_1, x_2, \ldots, x_N\} \in \mathbb{R}^d$. These could be
  - Feature points associated with the nodes of a graph.
  - Set of coordinates for a point-cloud in high-dimensional space.
- A scalar function is defined on these coordinates, $f: \mathbf{X} \rightarrow \mathbb{R}$, our signal to process is $\mathbf{f} = [f_1, f_2, \ldots, f_N]$.
- We may regard this dataset as a set of samples taken from a high-dimensional function $f(x): \mathbb{R}^d \rightarrow \mathbb{R}$.
- Key assumption – A distance-measure $w(x_i, x_j)$ between points in $\mathbb{R}^d$ is available to us. The function behind the scene is "regular":



Small $w(x_i, x_j)$ implies small $|f(x_i) - f(x_j)|$ for almost every pair (i, j)

Our Goal

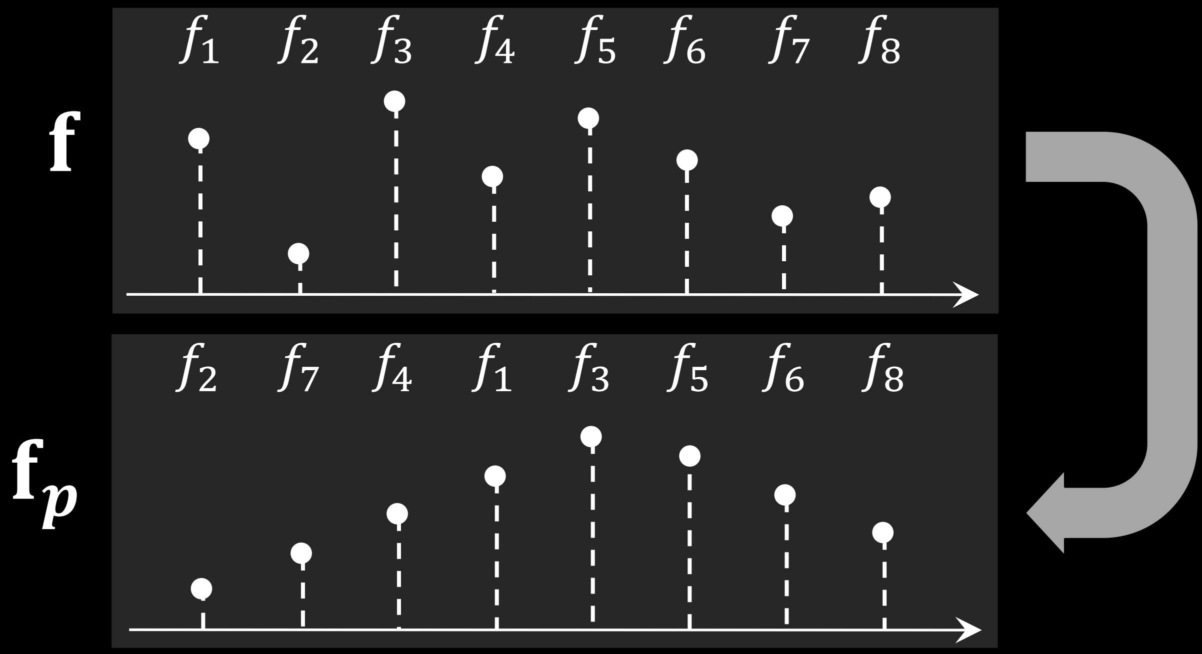


- We develop both an orthogonal wavelet transform and a redundant alternative, both efficiently representing the input signal $\mathbf{f}$.
- Our problem: The regular wavelet transform produces a small number of large coefficients when it is applied to piecewise regular signals. But, the signal (vector) $\mathbf{f}$ is not necessarily smooth in general.

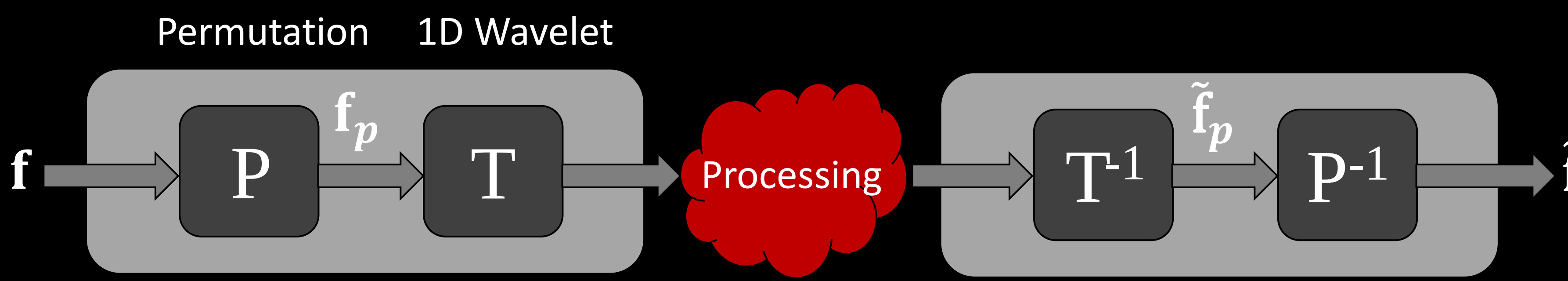
Previous and Related Work

- "Diffusion Wavelets"
  R. R. Coifman, and M. Maggioni, 2006.
- "Multiscale Methods for Data on Graphs and Irregular Multidimensional Situations"
  M. Jansen, G. P. Nason, and B. W. Silverman, 2008.
- "Wavelets on Graph via Spectal Graph Theory"
  D. K. Hammond, and P. Vandergheynst, and R. Gribonval, 2010.
- "Multiscale Wavelets on Trees, Graphs and High Dimensional Data: Theory and Applications to Semi Supervised Learning"
  M . Gavish, and B. Nadler, and R. R. Coifman, 2010.
- "Wavelet Shrinkage on Paths for Denoising of Scattered Data"
  D. Heinen and G. Plonka, to appear

The Main Idea (1) - Permutation

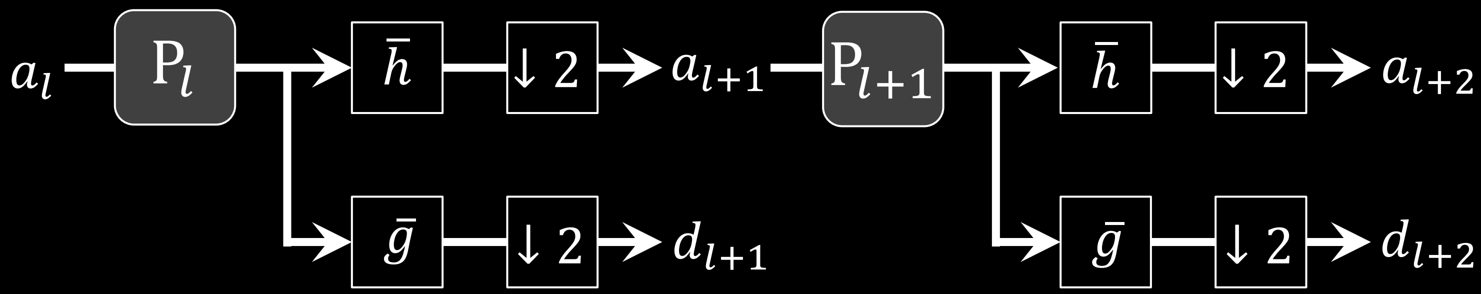


Permutation using
$\mathbf{X} = \{x_1, x_2, \ldots, x_N\}$



The Main Idea (2) - Permutation

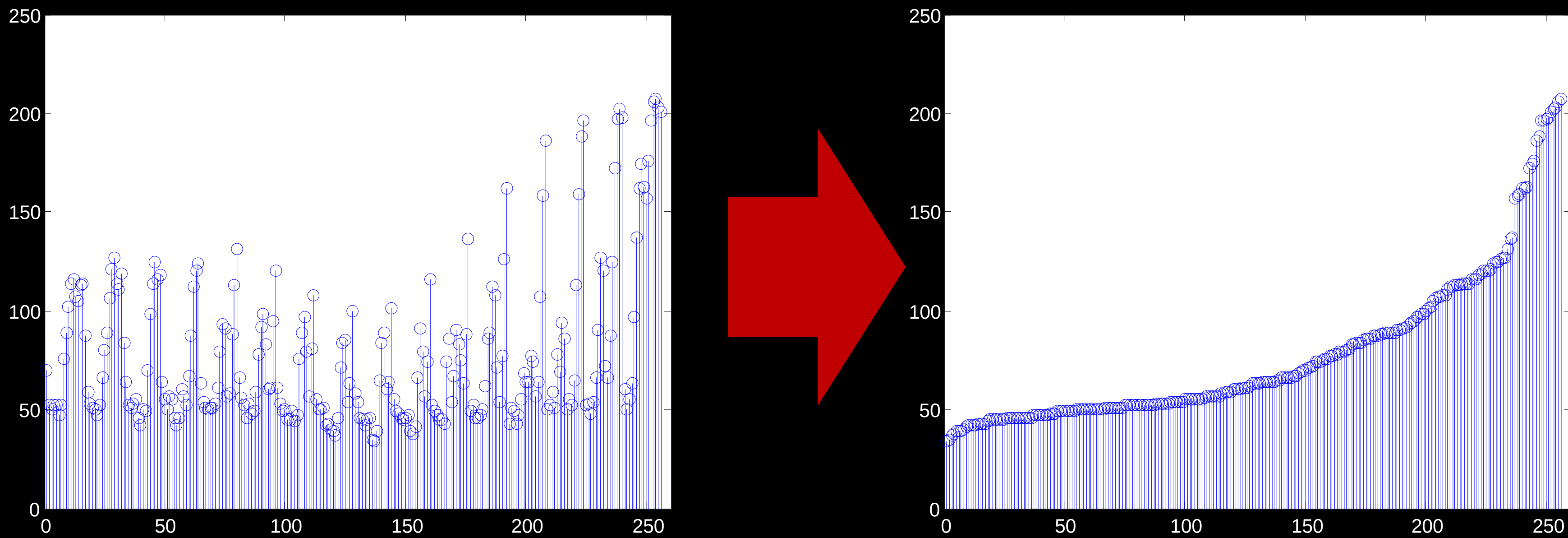
- In fact, we propose to perform a **different** permutation in each resolution level of the multi-scale pyramid:



- Naturally, these permutations will be applied reversely in the inverse transform.
- Thus, the difference between this and the plain 1D wavelet transform applied on $\mathbf{f}$ are the additional permutations, thus preserving the transform's **linearity** and unitarity, while also adapting to the input signal.

Building the Permutations (1)

- Lets start with P_0 – the permutation applied on the incoming signal.
- Recall: the wavelet transform is most effective for piecewise regular signals.
  $\longrightarrow$ thus, P_0 should be chosen such that $\mathrm{P}_0\mathbf{f}$ is most “regular”.
- So, … for example, we can simply permute by sorting the signal $\mathbf{f}$ …



Building the Permutations (2)

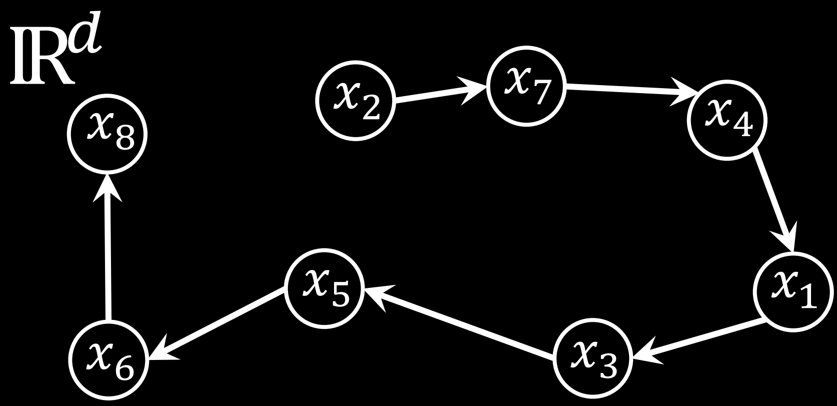
- **However**: we will be dealing with corrupted signals $\mathbf{f}$ (noisy, missing values, …) and thus such a sort operation is impossible.
- To our help comes the feature vectors in $\mathbf{X}$, which reflect on the order of the signal values, f_{k}. Recall:

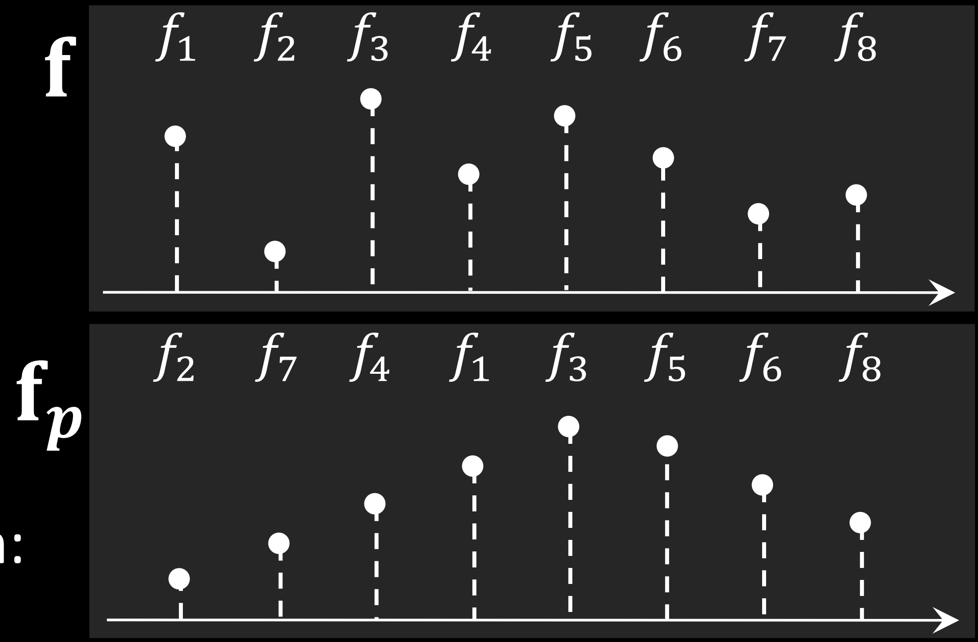
Small $w(x_i, x_j)$ implies small $|f(x_i) - f(x_j)|$ for almost every pair (i, j)

- Thus, instead of solving for the optimal permutation that "simplifies" $\mathbf{f}$, we order the features in $\mathbf{X}$ to the shortest path that visits in each point once, in what will be an instance of the Traveling-Salesman-Problem (TSP):

$$\min_{\mathrm{P}} \sum_{i=2}^{N} |f^p(i) - f^p(i-1)| \Rightarrow \min_{\mathrm{P}} \sum_{i=2}^{N} w(x_i^p, x_{i-1}^p)$$

Building the Permutations (3)



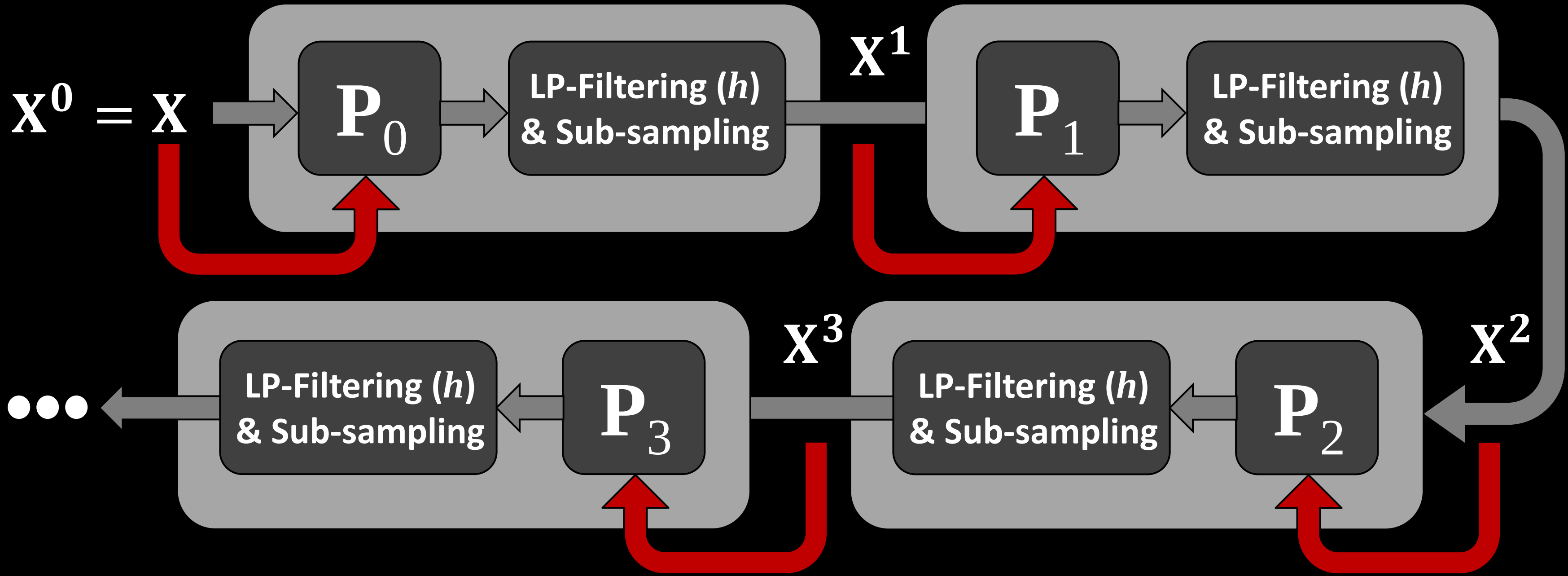


We handle the TSP task by a simple (and crude) approximation:

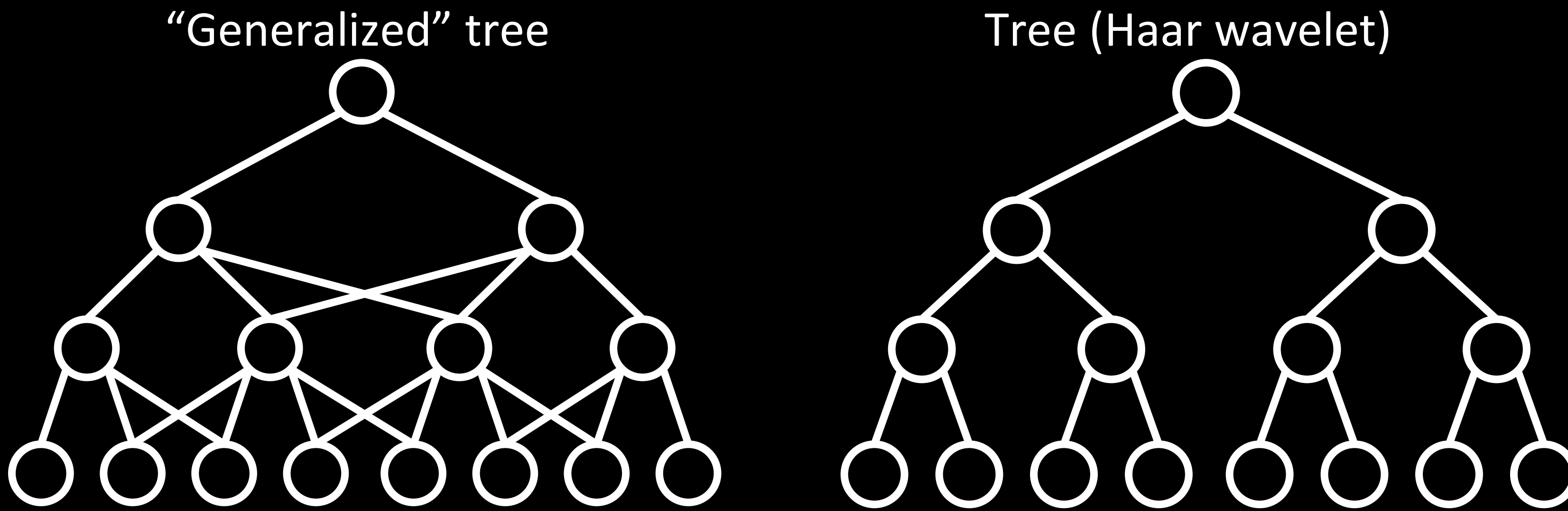
- Initialize with an arbitrary index j;
- Initialize the set of chosen indices to $\Omega(1)=\{j\}$;
- Repeat $k=1:1:N$-1 times:
  - Find x_i – the nearest neighbor to $x_{\Omega(\text{k})}$ such that $i \notin \Omega$;
  - Set $\Omega(\text{k}+1)=\{i\}$;
- Result: the set Ω holds the proposed ordering.

Building the Permutations (4)

- So far we concentrated on P_0 at the finest level of the multi-scale pyramid.
- In order to construct $\mathrm{P}_1, \mathrm{P}_2, \ldots, \mathrm{P}_{L\text{-}1}$, the permutations at the other pyramid's levels, we use the same method, applied on propagated (reordered, filtered and sub-sampled) feature-vectors through the same wavelet pyramid:



Why "Generalized Tree …"?



- Our proposed transform: **Generalized Tree-Based Wavelet Transform (GTBWT)**.
- We also developed a redundant version of this transform based on the stationary wavelet transform [Shensa, 1992] [Beylkin, 1992] – also related to the "A-Trous Wavelet" (will not be presented here).
- At this stage we could (or should) show how this works on point clouds/graphs, but we will take a different route and demonstrate these tools for images.

Part II – Handling Images

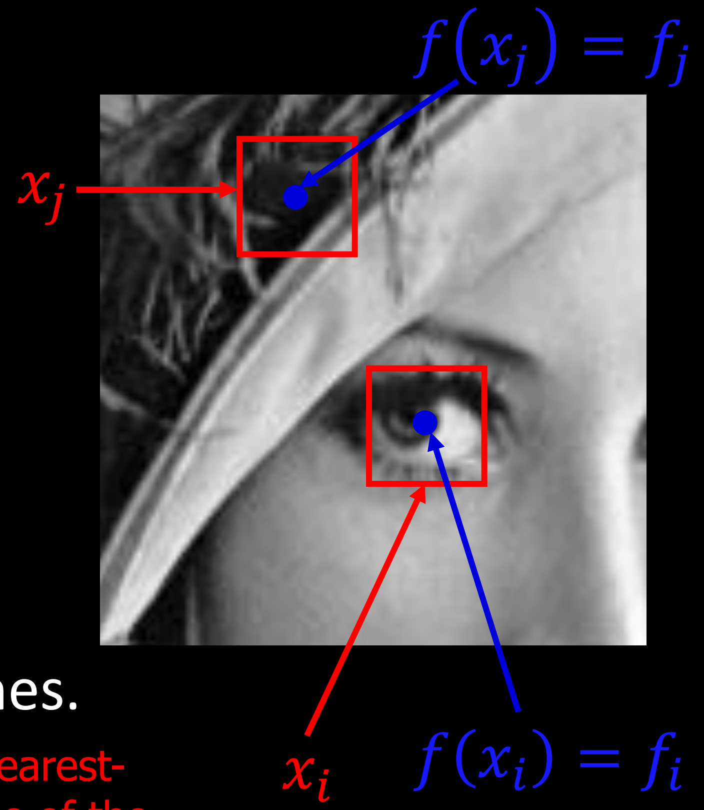
Using GTBWT by Handling Image Patches

This part is taken from the same papers mentioned before …

- I. Ram, M. Elad, and I. Cohen, "Generalized Tree-Based Wavelet Transform", IEEE Trans. Signal Processing, vol. 59, no. 9, pp. 4199–4209, 2011.
- I. Ram, M. Elad, and I. Cohen, "Redundant Wavelets on Graphs and High Dimensional Data Clouds", IEEE Signal Processing Letters, Vol. 19, No. 5, pp. 291–294 , May 2012.

Could Images Fit This Data-Structure?

- Yes. Starting with an image of size $\sqrt{N} \times \sqrt{N}$, do the following:
  - Extract all possible patches of size $\sqrt{d} \times \sqrt{d}$ with complete overlaps – these will serve as the set of features (or coordinates) matrix $\mathbf{X}$.
  - The values $f(x_i) = f_i$ will be the center pixel in these patches.



- Once constructed this way, we forget all about spatial proximities in the image*, and start thinking in terms of (Euclidean) proximities between patches.

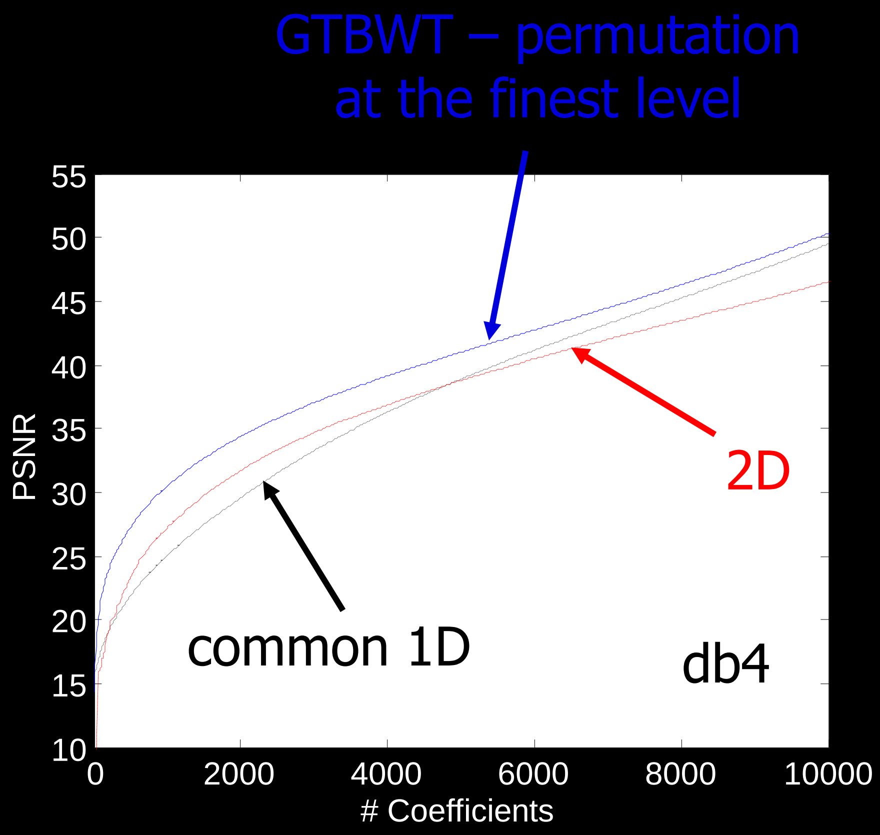
* Not exactly. Actually, if we search the nearest-neighbors within a limited window, some of the spatial proximity remains.

Lets Try (1)

For a 128×128 center portion of the image Lenna, we compare the image representation efficiency of the

- ❑ GTBWT
- ❑ A common 1D wavelet transform
- ❑ 2D wavelet transform.

We measure efficiency by the m-term approximation error, i.e. reconstructing the image from m largest coefficients, zeroing the rest.

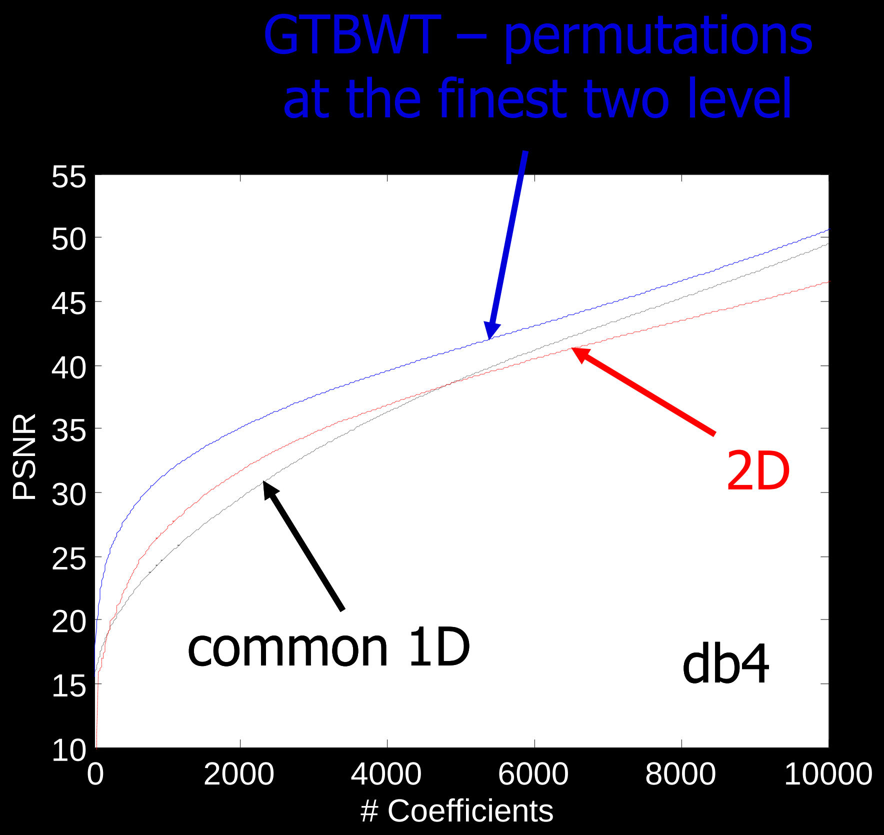


Lets Try (2)

For a 128×128 center portion of the image Lenna, we compare the image representation efficiency of the

- GTBWT
- A common 1D wavelet transform
- 2D wavelet transform.

We measure efficiency by the m-term approximation error, i.e. reconstructing the image from m largest coefficients, zeroing the rest.



Lets Try (3)

For a 128×128 center portion of the image Lenna, we compare the image representation efficiency of the

- GTBWT
- A common 1D wavelet transform
- 2D wavelet transform.

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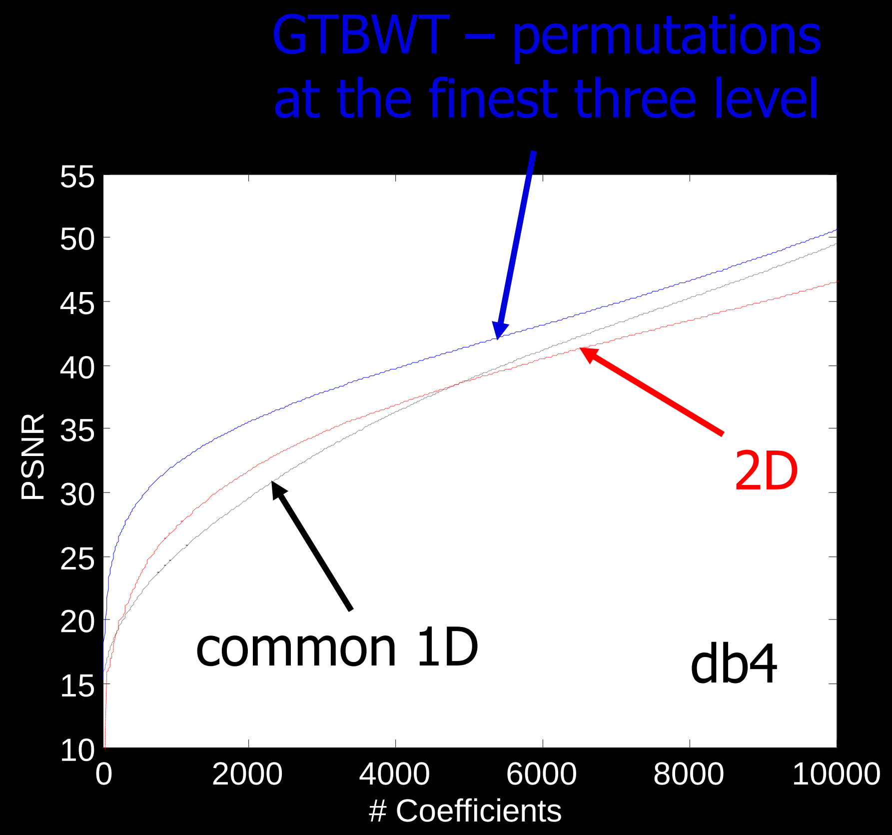

Lets Try (4)

For a 128×128 center portion of the image Lenna, we compare the image representation efficiency of the

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- A common 1D wavelet transform
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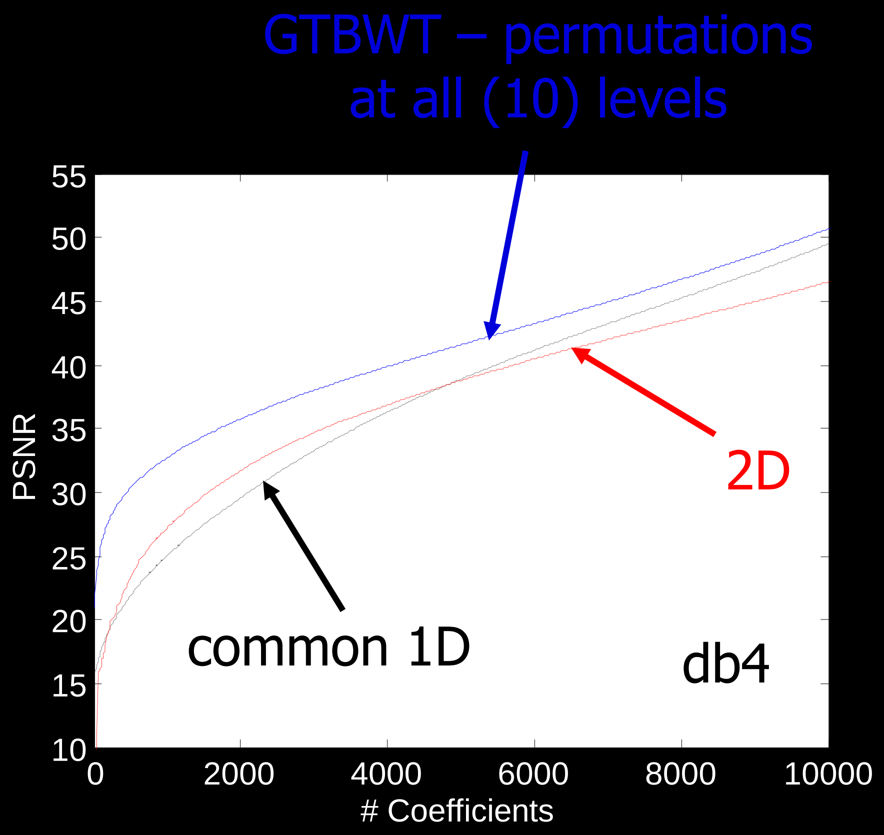
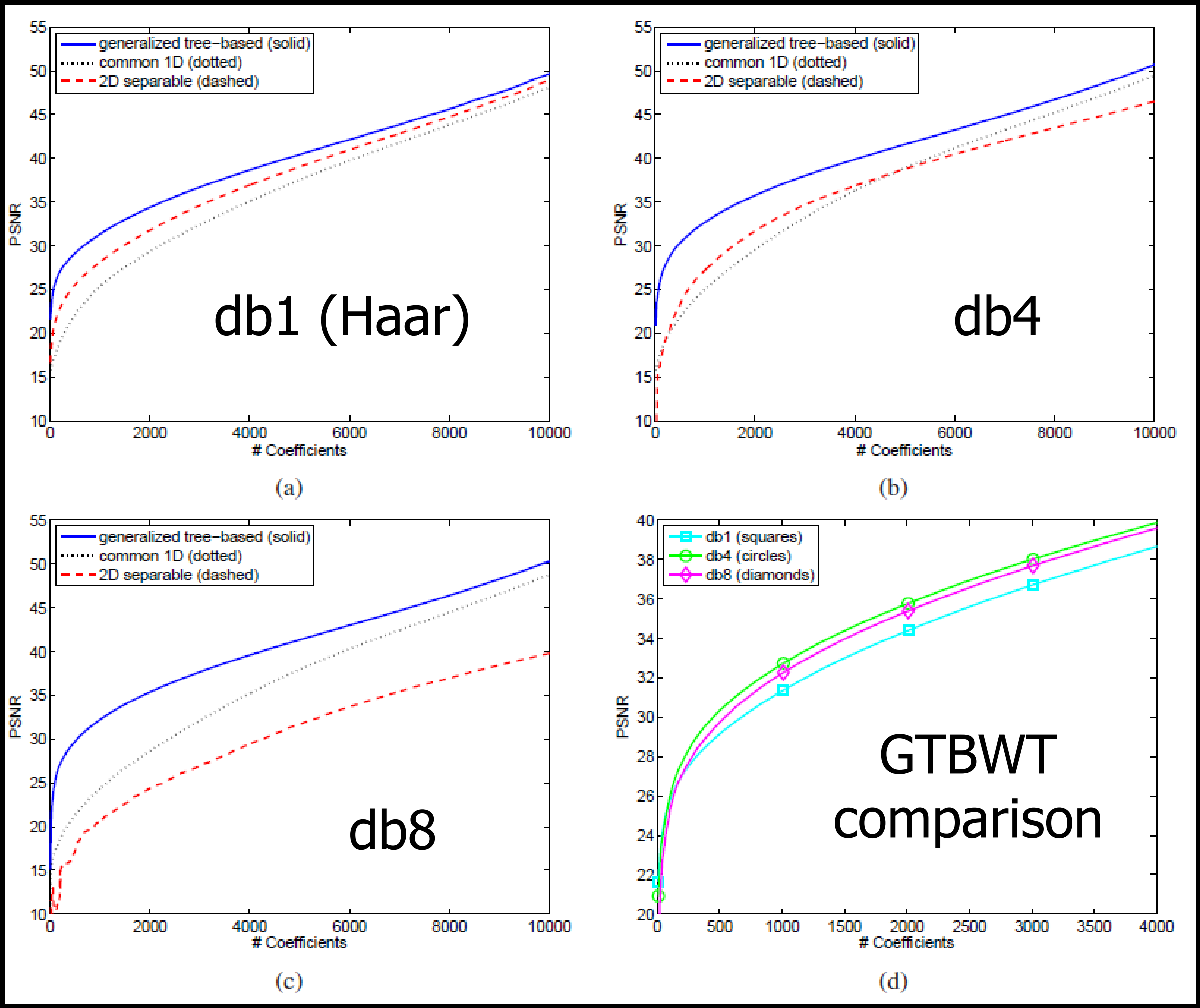
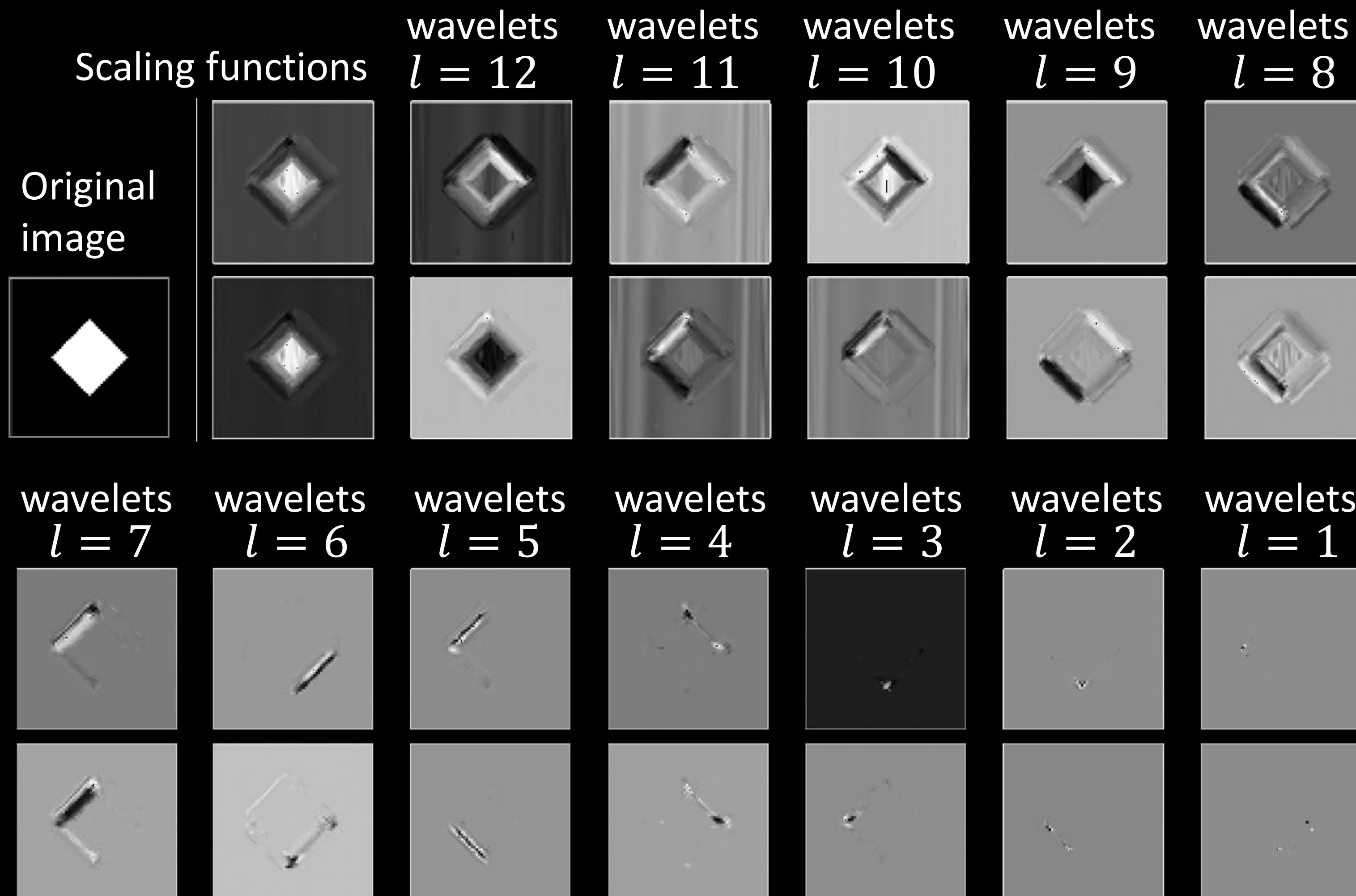


Comparison Between Different Wavelets



The Representation's Atoms – Synthetic Image



The Representation's Atoms – Lenna

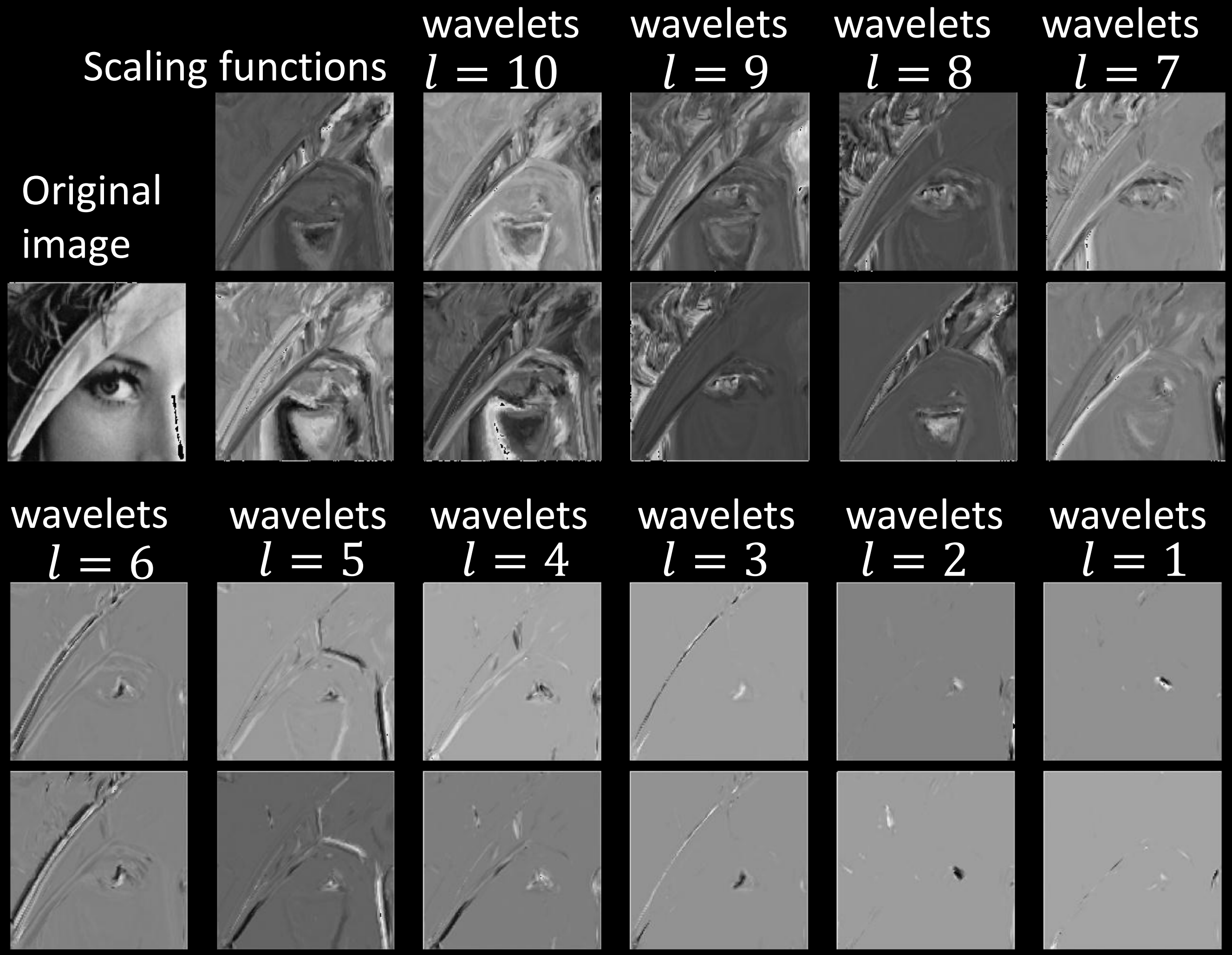


Image Denoising using GTBWT

- We assume that the noisy image, $\tilde{F}$, is a noisy version of a clean image, F, contaminated by white zero-mean Gaussian additive noise with known STD=σ.
- The vectors $\tilde{\mathbf{f}}$ and $\mathbf{f}$ are lexicographic ordering of the noisy and clean images.
- Our goal: recover $\mathbf{f}$ from $\tilde{\mathbf{f}}$, and we will do this using shrinkage over GTBWT:
  - We extract all patches from the noisy image as described above;
  - We apply the GTBWT on this data set;
  - The wavelet coefficients obtained go through a shrinkage operation; and
  - We transform back to get the final outcome.

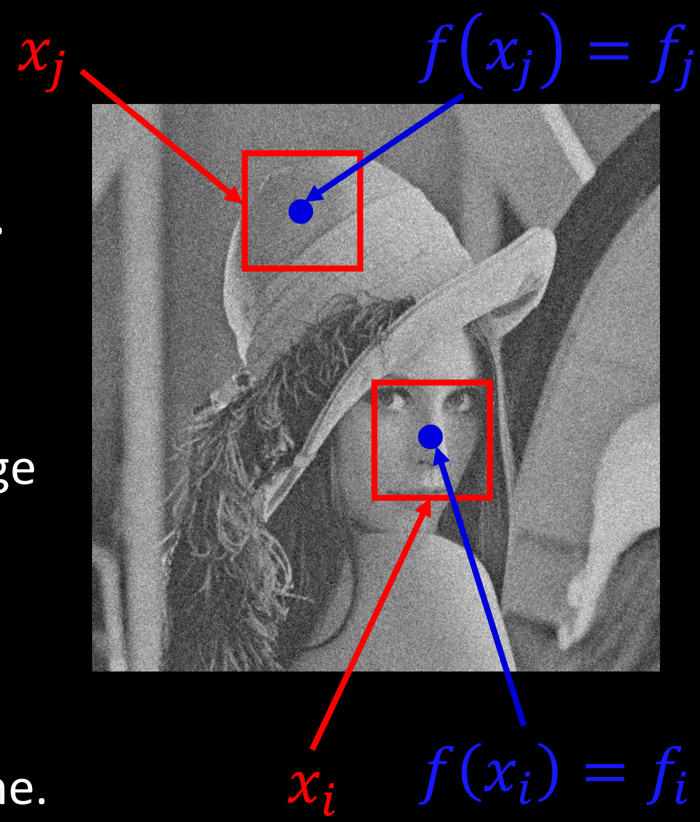


Image Denoising – Block-Diagram

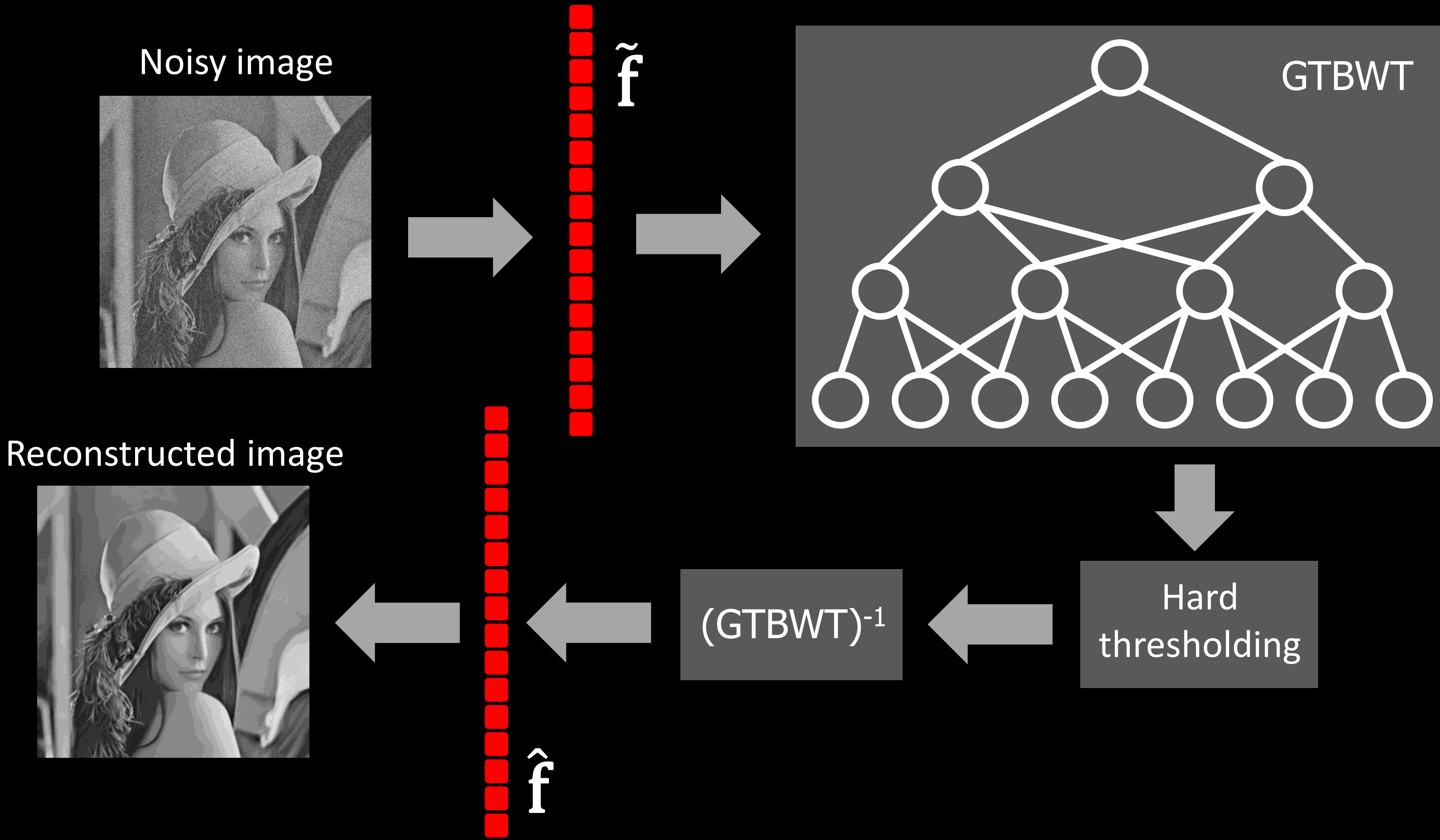


Image Denoising – Improvements

Cycle-spinning: Apply the above scheme several (10) times, with a different GTBWT (different random ordering), and average.

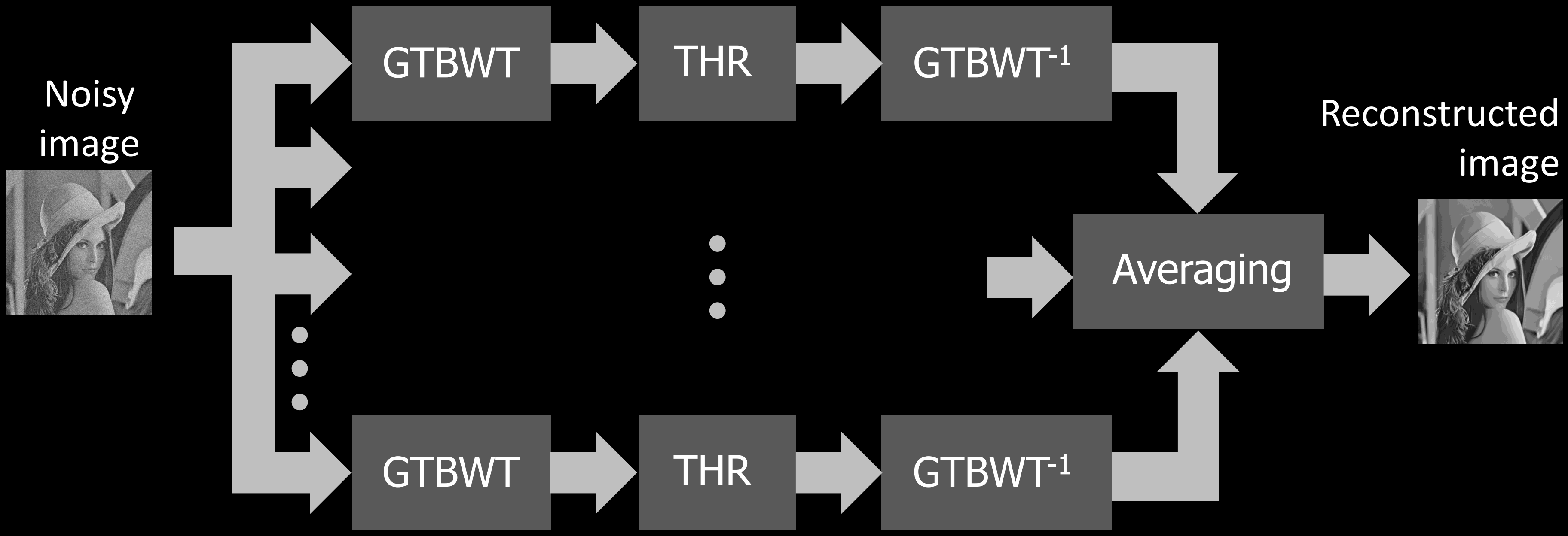


Image Denoising – Improvements

Sub-image averaging: A by-product of GTBWT is the propagation of the whole patches. Thus, we get n transform vectors, each for a shifted version of the image and those can be averaged.

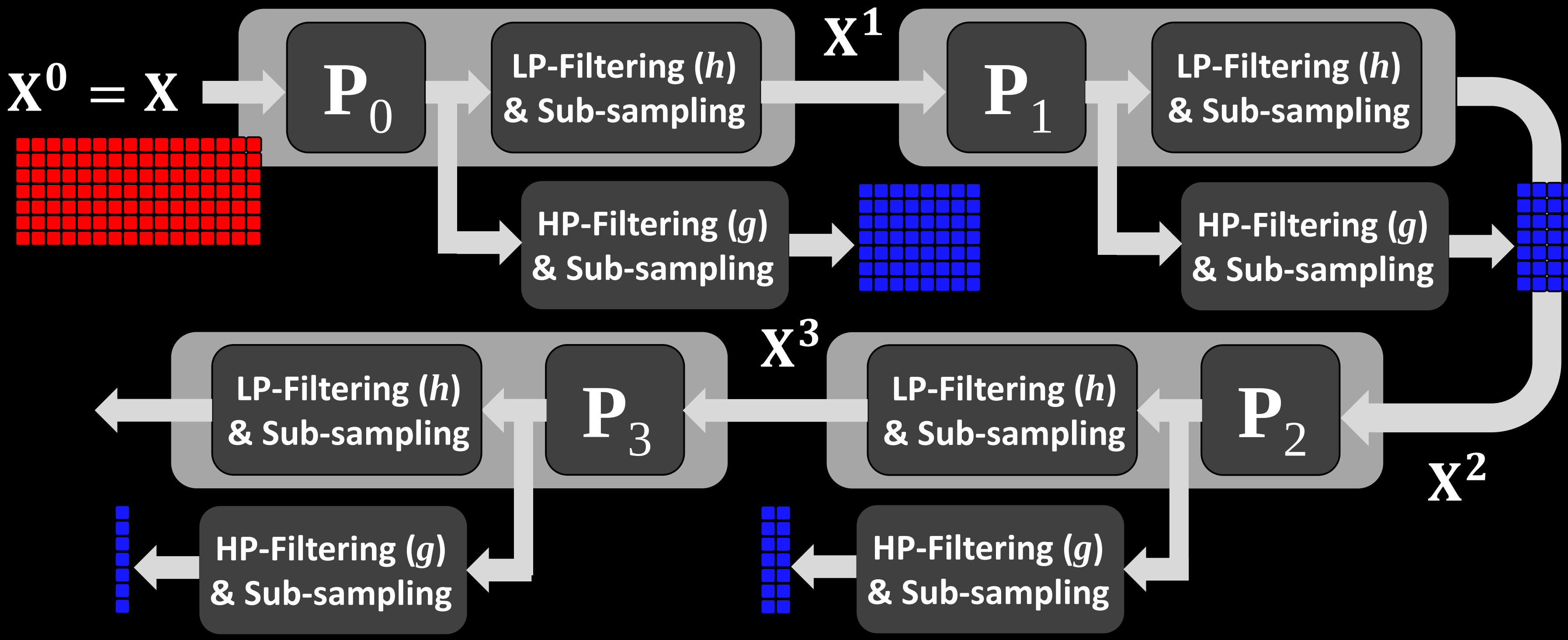


Image Denoising – Improvements

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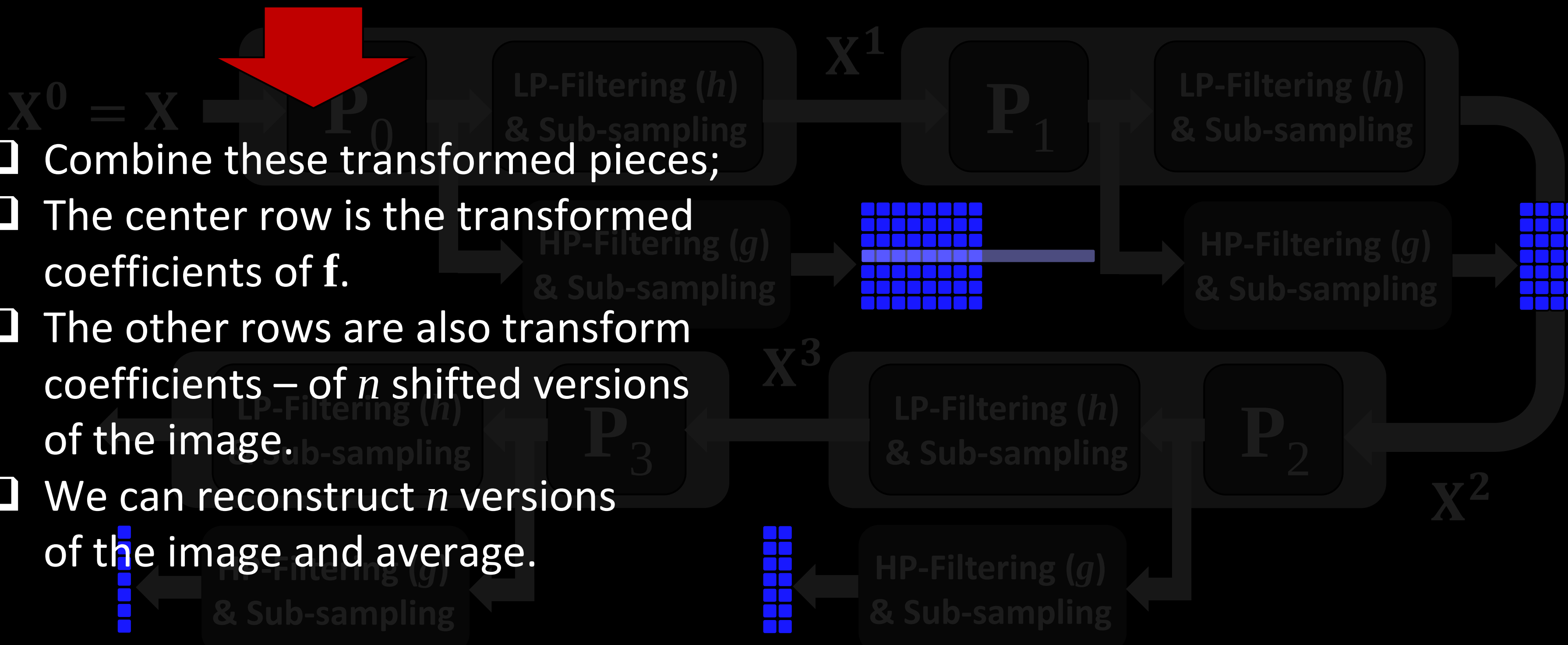


- Combine these transformed pieces;
- The center row is the transformed coefficients of $\mathbf{f}$.
- The other rows are also transform coefficients – of n shifted versions of the image.
- We can reconstruct n versions of the image and average.

Image Denoising – Improvements

Restricting the NN: It appears that when searching the nearest-neighbor for the ordering, restriction to near-by area is helpful, both computationally (obviously) and in terms of the output quality.

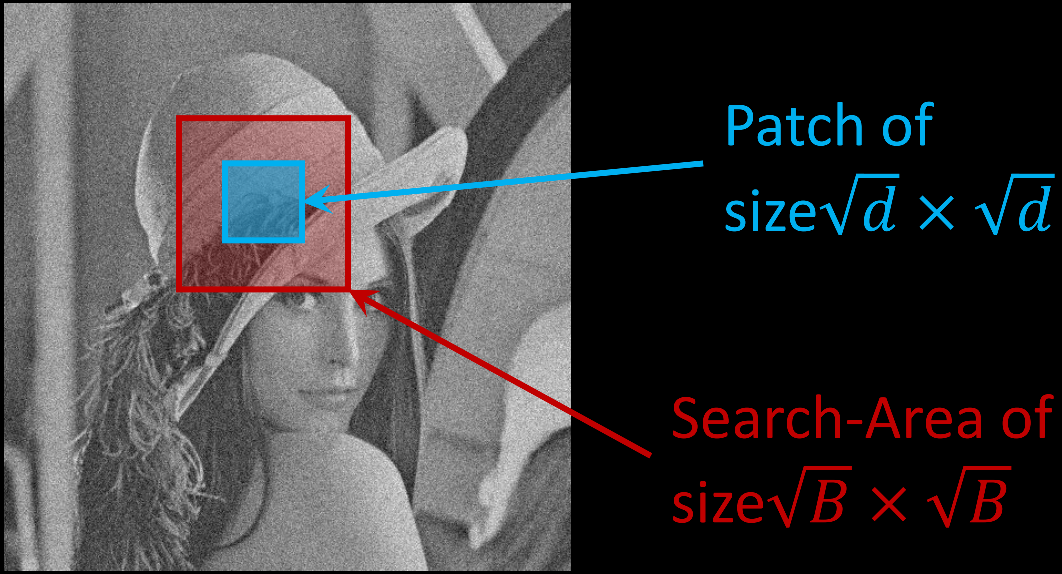
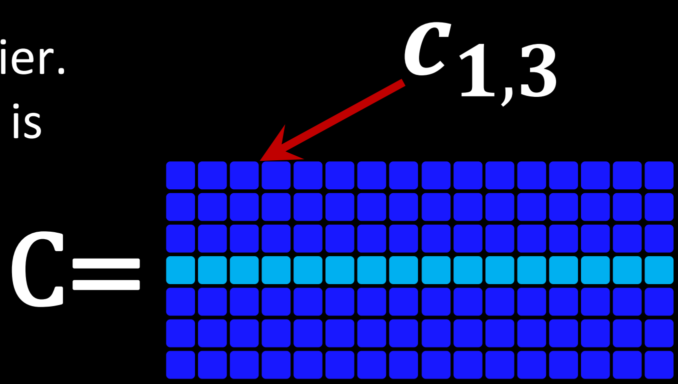


Image Denoising – Improvements

Improved thresholding: Instead of thresholding the wavelet coefficients based on their value, threshold them based on the norm of the (transformed) vector they belong to:

- Recall the transformed vectors as described earlier.
- Classical thresholding: every coefficient within $\mathbf{C}$ is passed through the function:

$$c_{i,j} = \begin{cases} c_{i,j} & |c_{i,j}| \geq T \\ 0 & |c_{i,j}| < T \end{cases}$$



- The proposed alternative would be to force "joint-sparsity" on the above array of coefficients, forcing all rows to share the same support:

$$c_{i,j} = \begin{cases} c_{i,j} & \|c_{*,j}\|_2 \geq T \\ 0 & \|c_{*,j}\|_2 < T \end{cases}$$

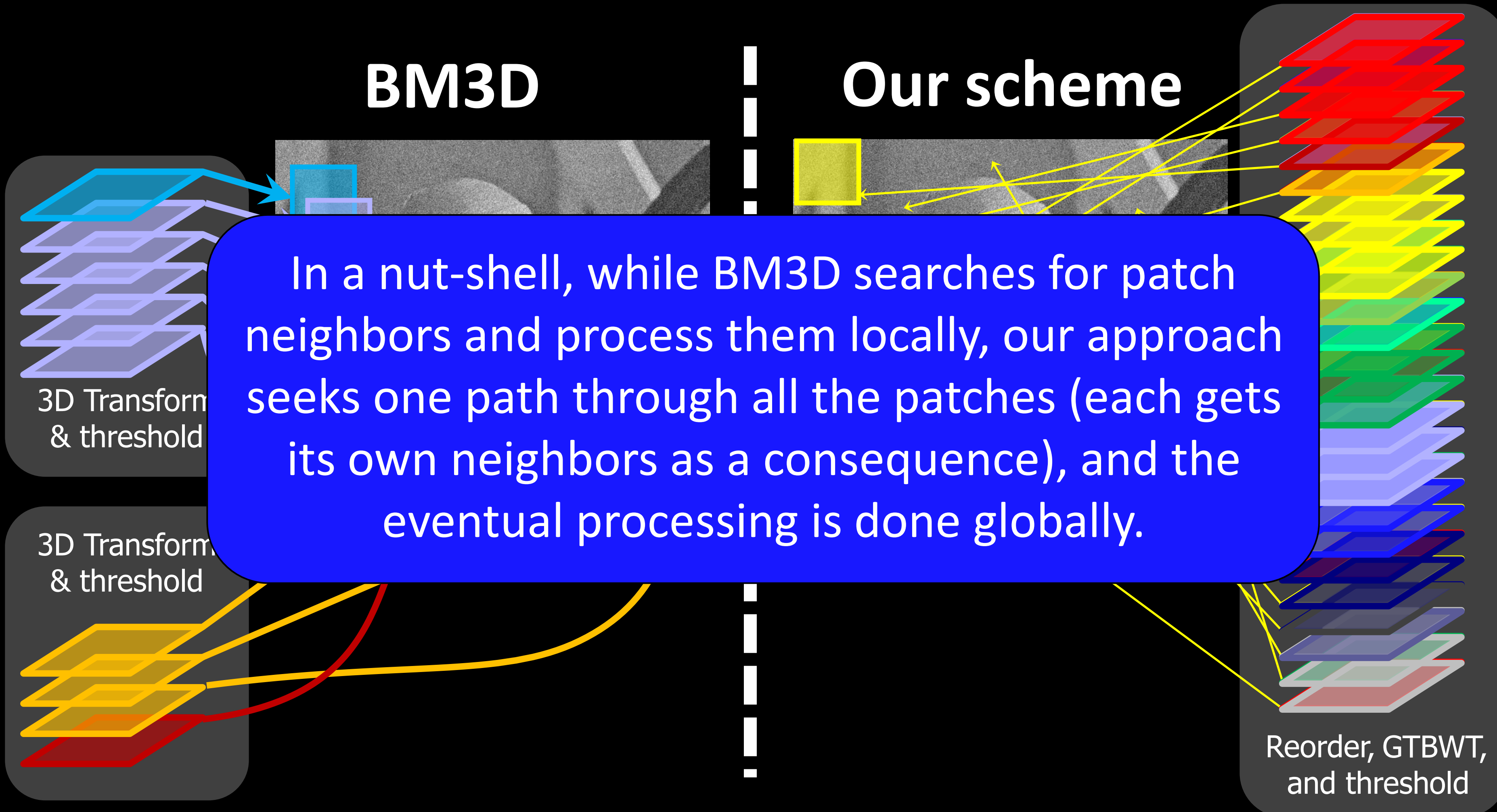
Image Denoising – Results

- We apply the proposed scheme with the Symmlet 8 wavelet to noisy versions of the images Lena and Barbara
- For comparison reasons, we also apply to the two images the K-SVD and BM3D algorithms.

σ/PSNR	Image	K-SVD	BM3D	GTBWT
10/28.14	Lena	35.51	35.93	35.87
	Barbara	34.44	34.98	34.94
25/20.18	Lena	31.36	32.08	32.16
	Barbara	29.57	30.72	30.75

- The PSNR results are quite good and competitive.
- What about run time?

Relation to BM3D?



In a nut-shell, while BM3D searches for patch neighbors and process them locally, our approach seeks one path through all the patches (each gets its own neighbors as a consequence), and the eventual processing is done globally.

Part III – Frame

Interpreting the GTBWT as a Frame and using it as a Regularizer

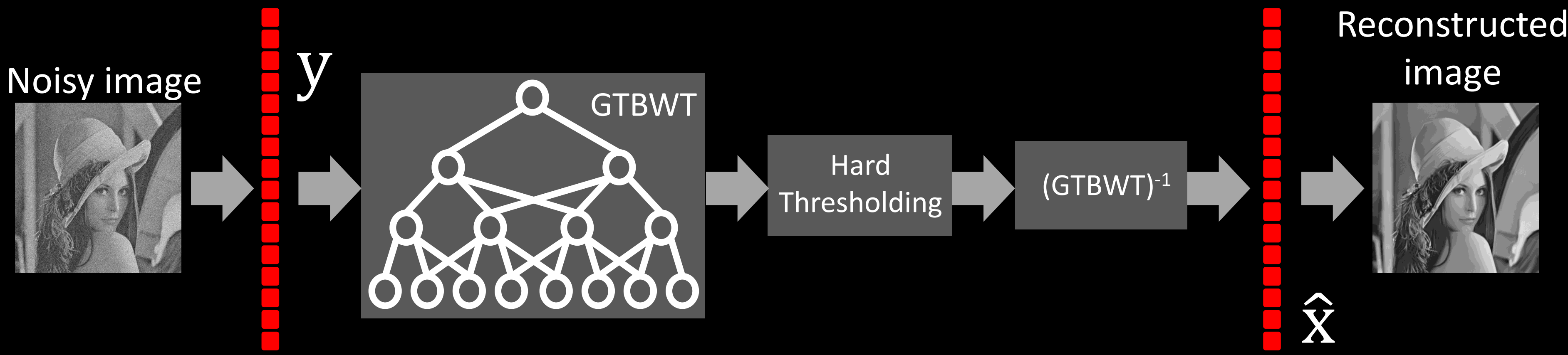
This part is documented in the following draft:

- I. Ram, M. Elad, and I. Cohen, "The RTBWT Frame – Theory and Use for Images", working draft to be submitted soon.

We rely heavily on

- Danielyan, Katkovnik, and Eigiazarian, "BM3D frames and Variational Image Deblurring", IEEE Trans. on Image Processing, Vol. 21, No. 4, pp. 1715-1728, April 2012.

Recall Our Core Scheme



Or, put differently, $\hat{x} = \mathbf{D} \cdot T\{\boldsymbol{\Omega} y\}$: We refer to GTBWT as a redundant frame, and use a "heuristic" shrinkage method with it, which aims to approximate the solution of

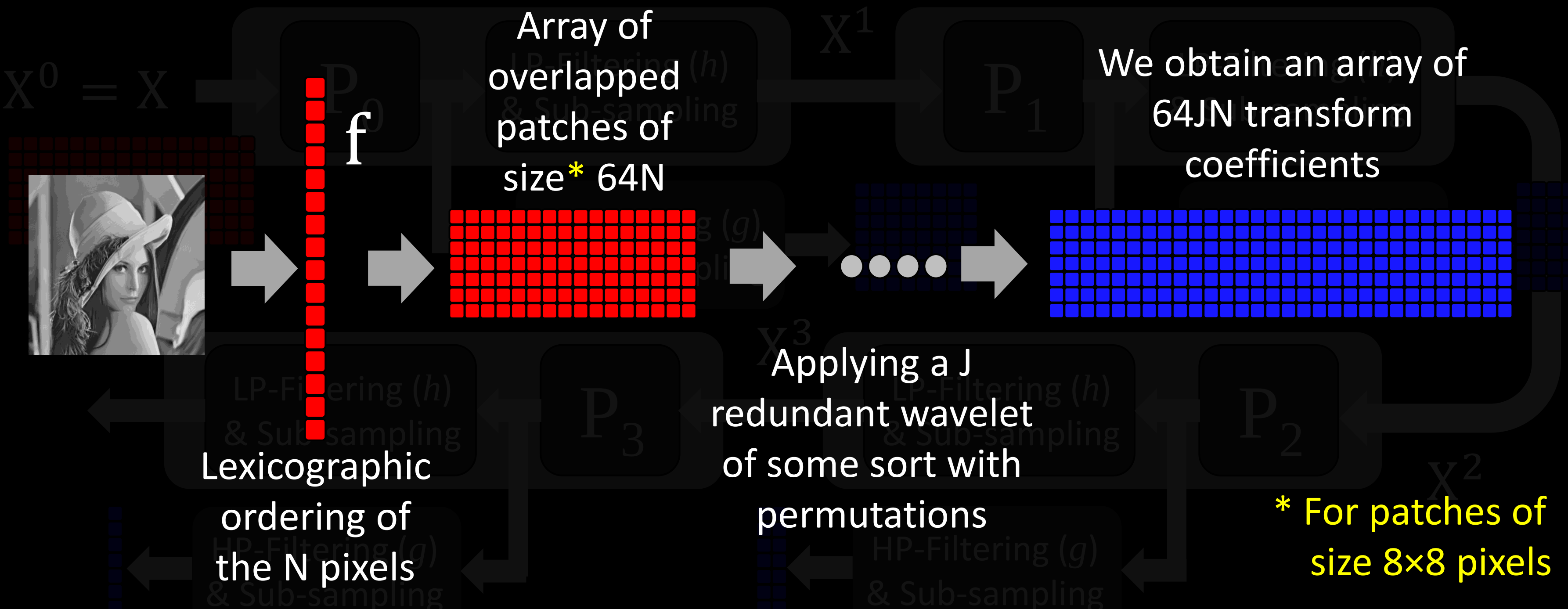
Synthesis: $$\hat{x} = \mathbf{D} \cdot \underset{\alpha}{\text{Argmin}} \|\mathbf{D}\alpha - y\|_2^2 + \lambda \|\alpha\|_p^p$$

or

Analysis: $$\hat{x} = \underset{f}{\text{Argmin}} \|x - y\|_2^2 + \lambda \|\boldsymbol{\Omega} x\|_p^p$$

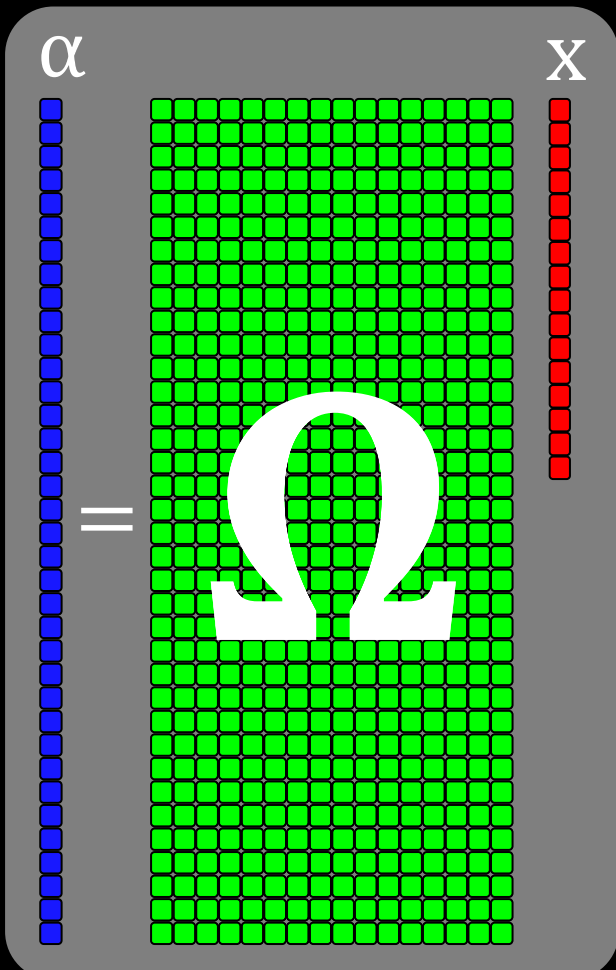
So, Who is Our Frame?

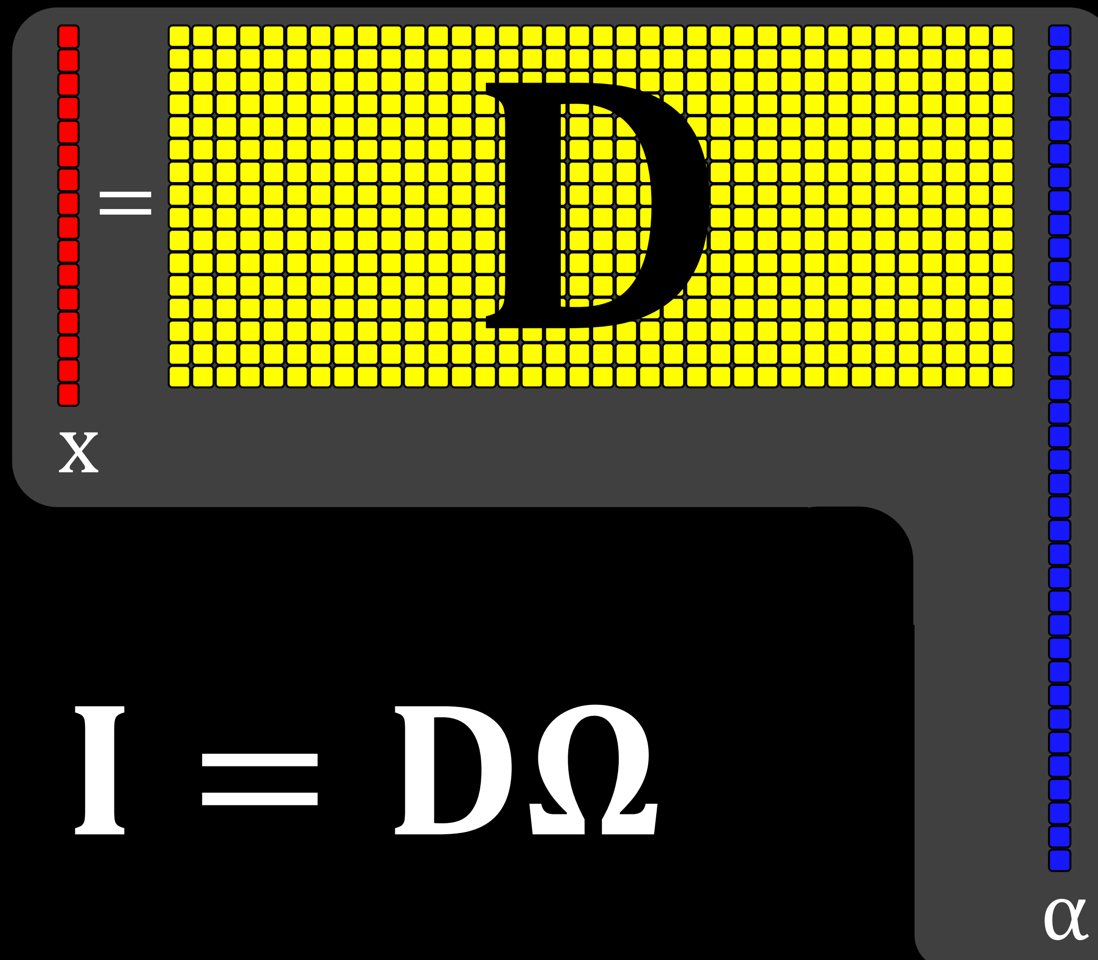
Bottom line:



⇒ The redundancy of our frame is 64J

Our Notations





$$\mathbf{I} = \mathbf{D}\boldsymbol{\Omega}$$

What Can We Do With This Frame?

We could solve various inverse problems of the form:

$$\mathrm{y} = \mathbf{A}\mathrm{x} + \mathrm{v}$$

where: x is the original image
v is an AWGN, and
$\mathbf{A}$ is a degradation operator of any sort

We could consider the synthesis, the analysis, or their combination:

$$\{\hat{\mathrm{x}}, \hat{\alpha}\} = \underset{\alpha,\mathrm{x}}{\mathrm{Argmin}} \begin{array}{c} \|\mathrm{y} - \mathbf{A}\mathrm{x}\|_2^2 + \frac{1}{\beta}\|\mathbf{D}\alpha - \mathrm{x}\|_2^2 + \\ +\lambda\|\alpha\|_p^p + \frac{1}{\mu}\|\mathbf{\Omega}\mathrm{x} - \alpha\|_2^2 \end{array}$$

$\beta = 0$, $\mu = \infty$ $\rightarrow$ Synthesis

$\beta = \infty$, $\mu = 0$ $\rightarrow$ Analysis

Generalized Nash Equilibrium*

Instead of minimizing the joint analysis/synthesis problem:

$$\{\hat{x},\hat{\alpha}\} = \underset{\alpha,x}{\text{Argmin}}\ \|y - \mathbf{A}x\|_2^2 + \frac{1}{\beta}\|\mathbf{D}\alpha - x\|_2^2 + +\lambda\|\alpha\|_p^p + \frac{1}{\mu}\|\mathbf{\Omega}x - \alpha\|_2^2$$

break it down into two separate and easy to handle parts:

and solve iteratively

$$x_{k+1} = \underset{x}{\text{Argmin}}\ \|y - \mathbf{A}x\|_2^2 + \frac{1}{\beta}\|\mathbf{D}\alpha_k - x\|_2^2$$

$$\alpha_{k+1} = \underset{\alpha}{\text{Argmin}}\ \lambda\|\alpha\|_p^p + \frac{1}{\mu}\|\mathbf{\Omega}x_{k+1} - \alpha\|_2^2$$

* Danielyan, Katkovnik, and Eigiazarian, "BM3D frames and Variational Image Deblurring", IEEE Trans. on Image Processing, Vol. 21, No. 4, pp. 1715-1728, April 2012.

Deblurring Results

Original

Blurred

Restored

Deblurring Results

Image	Input PSNR	BM3D-DEB ISNR	IDD-BM3D ISNR init. with BM3D-DEB	Ours ISNR Init. with BM3D-DEB	Ours ISNR 3 iterations with simple initialization
Lena	27.25	7.95	7.97	8.08	8.20
Barbara	23.34	7.80	7.64	8.25	6.21
House	25.61	9.32	9.95	9.80	10.06
Cameraman	22.23	819	8.85	9.19	8.52

Blur PSF = $\dfrac{1}{1 + i^2 + j^2} \quad -7 \leq i, j \leq 7$

σ^2=2

Part IV – Patch (Re)-Ordering

Lets Simplify Things, Shall We?

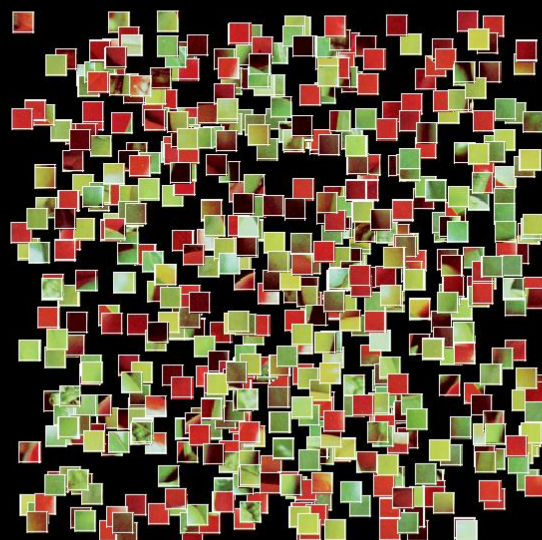
This part is based on the paper:

- I. Ram, M. Elad, and I. Cohen, "Image Processing using Smooth Ordering of its Patches", Submitted to IEEE Transactions on Image Processing.

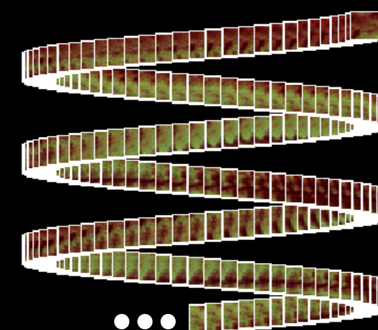
Returning to the Basics

Suppose we start with a clean image

We extract all (with overlaps) patches of size B×B (e.g. B=20)

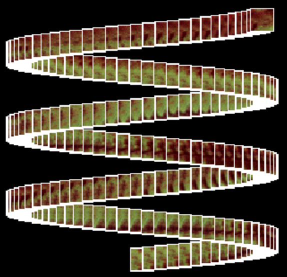


Then we order these patches to form the shortest path, as before

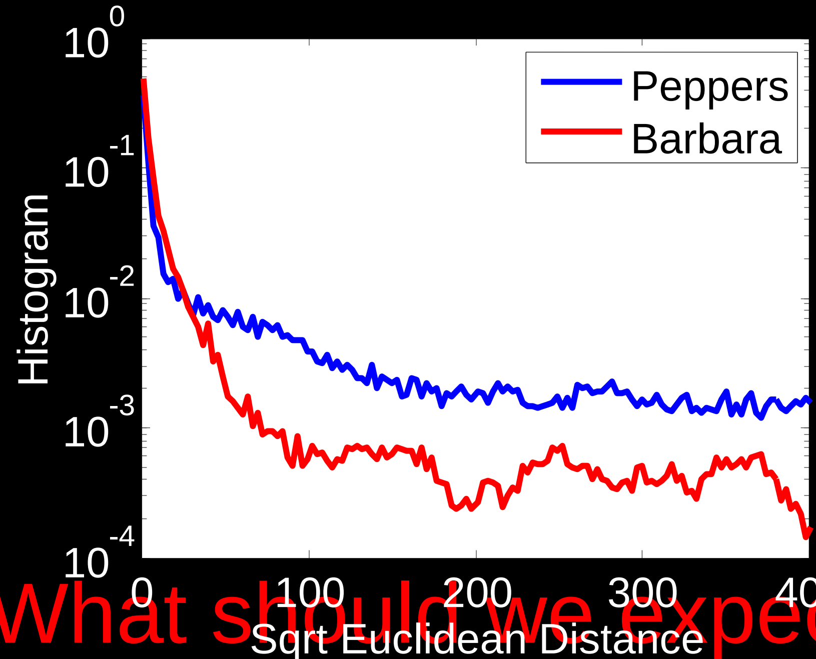


What should we expect?

Spatial Neighbor ≠ Euclidean Neighbor



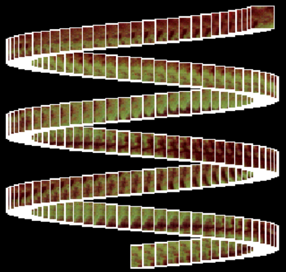
Spatial neighbors are not necessarily expected to remain neighbors in the new ordering



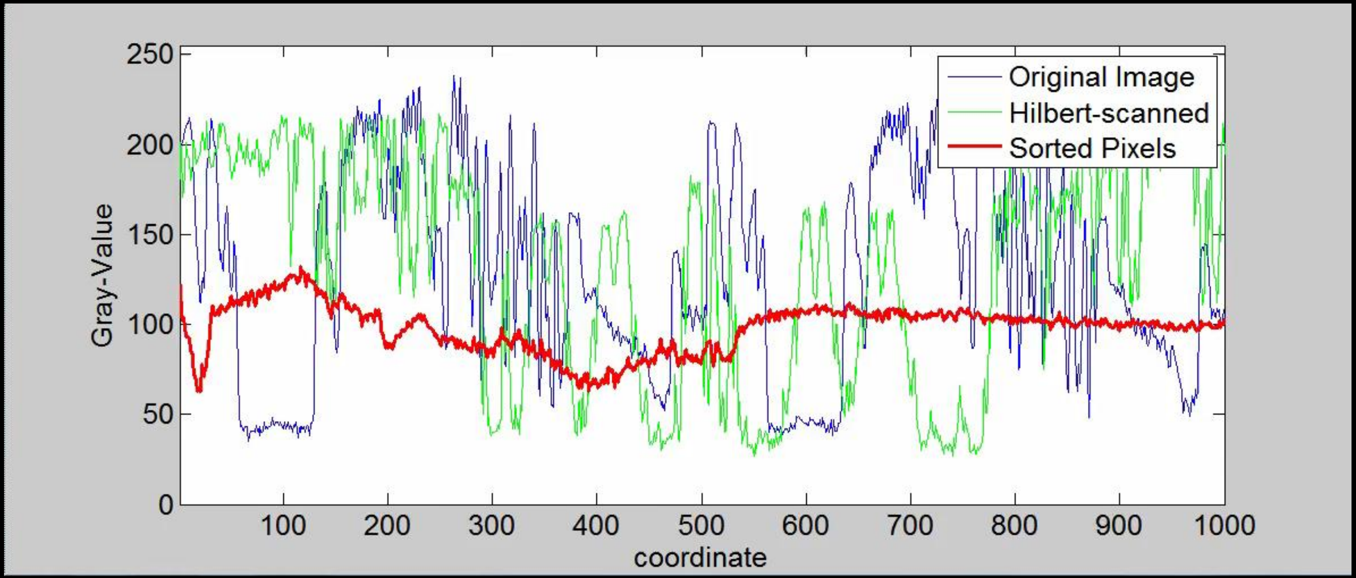
What should we expect?

The Reordered Signal is More Regular

What should we expect?



Considering the center (or any other) pixel in each patch, the new path is expected to lead to very smooth* (or at least, piece-wise smooth) 1D signal.



* Measure of smoothness:

$$\frac{1}{L}\sum_{k=2}^{L}|x[k]-x[k-1]|$$

1. Raster scan: 9.57
2. Hilbert curve: 11.77
3. Sorted (ours): 5.63

Processing the Permuted Pixels

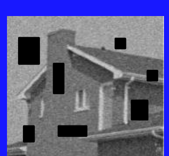
Assumptions:

- After a shortest-path reordering of the patches form a clean image, we expect a highly regular signal.
- Reordering a corrupted image is likely to lead to a good quality sort, due to the robustness brought by the patch-matching.

An Idea: Given a corrupted image of the form: $\mathbf{y} = \mathbf{M}\mathbf{x} + \mathbf{v}$

where: $\mathbf{x}$ is the original image
$\mathbf{v}$ is an AWGN, and
$\mathbf{M}$ is a point-wise degradation operator,

Apply this process:



Corrupted Image $\tilde{\mathbf{f}}$ → Re-order the pixels to a 1D signal → Process the 1D signal with a simple filter → Re-order the pixels back to their location →

Use the Reordering for Denoising

Noisy with σ=25 (20.18dB)

* This result is obtained with (i) cycle-spinning, (ii) sub-image averaging, (iii) two iterations, (iv) learning the filter , and (v) switched smoothing.

Extract all (with overlaps) patches of size 10×10

Order these patches as before (TSP)

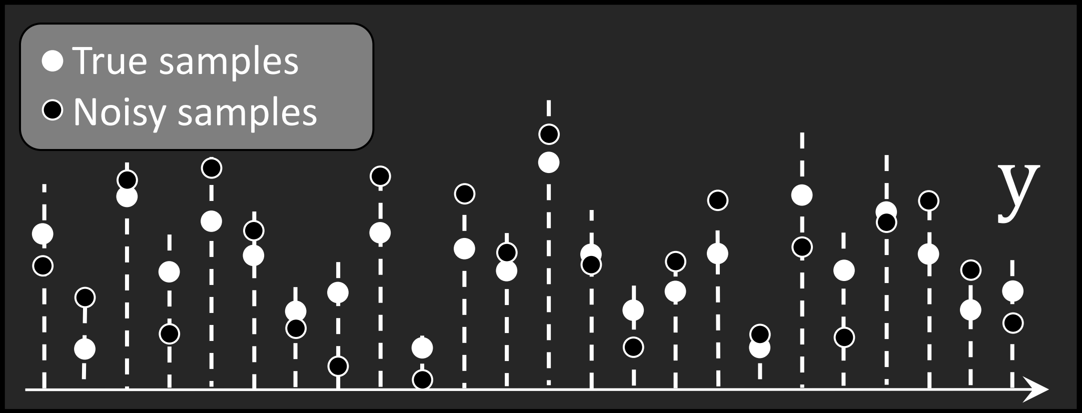
Take the center-row – it represents a permutation of the image pixels to a regular function

Smooth/filter the values along this row in a simple way

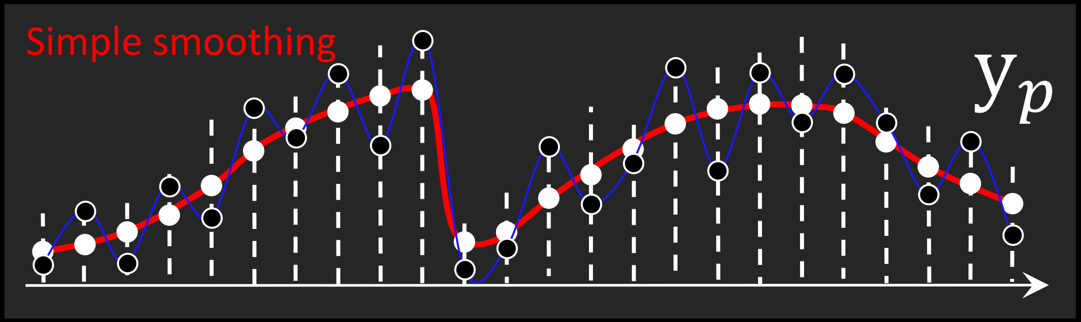
Reconstruction: 32.65dB*

Intuition: Why Should This Work?

Noisy with σ=25 (20.18dB)



Ordering based on the noisy pixels



Reconstruction: 32.65dB

The "Simple Smoothing" We Do

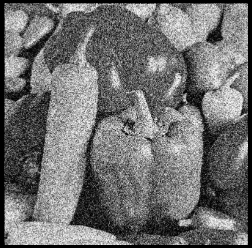
Simple smoothing works fine

but

We can do better by a training phase

Training image

Original image



Noisy image

Compute the TSP permutation

Apply the permutation on the pixels

Apply the permutation on the pixels

Apply a 1D filter h

+

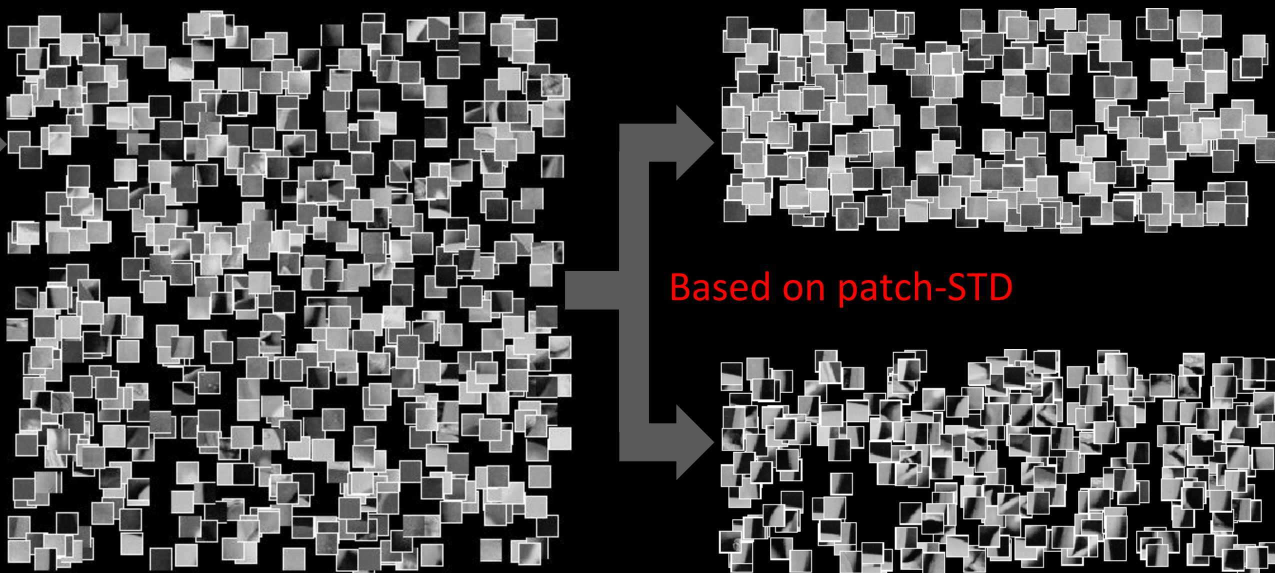
-

optimize h to minimize the reconstruction MSE

Naturally, this is done off-line and on other images

Filtering – A Further Improvement

Cluster the patches to smooth and textured sets, and train a filter per each separately



The results we show hereafter were obtained by:

(i) Cycle-spinning
(ii) Sub-image averaging
(iii) Two iterations
(iv) Learning the filter , and
(v) Switched smoothing.

Denoising Results Using Patch-Reordering

Image		σ/PSNR [dB]		
		10 / 28.14	25 / 20.18	50 / 14.16
Lena	K-SVD	35.49	31.36	27.82
	1st iteration	35.33	31.58	28.54
	2nd iteration	35.41	31.81	29.00
Barbara	K-SVD	34.41	29.53	25.40
	1st iteration	34.48	30.46	27.17
	2nd iteration	34.46	30.54	27.45
House	K-SVD	36.00	32.12	28.15
	1st iteration	35.58	32.48	29.37
	2nd iteration	35.94	32.65	29.93

Bottom line: (1) This idea works very well;
(2) It is especially competitive for high noise levels; and
(3) A second iteration almost always pays off.

What About Inpainting?

0.8 of the pixels are missing

Extract all (with overlaps) patches of size 9×9

Order these patches as before distance uses EXISTING pixels only

Take the center-row – it represents a permutation of the image pixels to a regular function

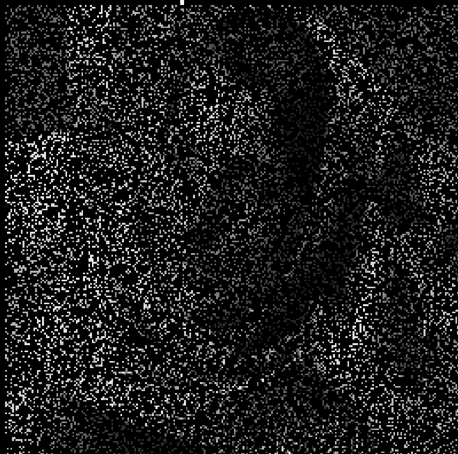
Fill the missing values in a simple (cubic interpolation) way

Reconstruction: 29.71dB *

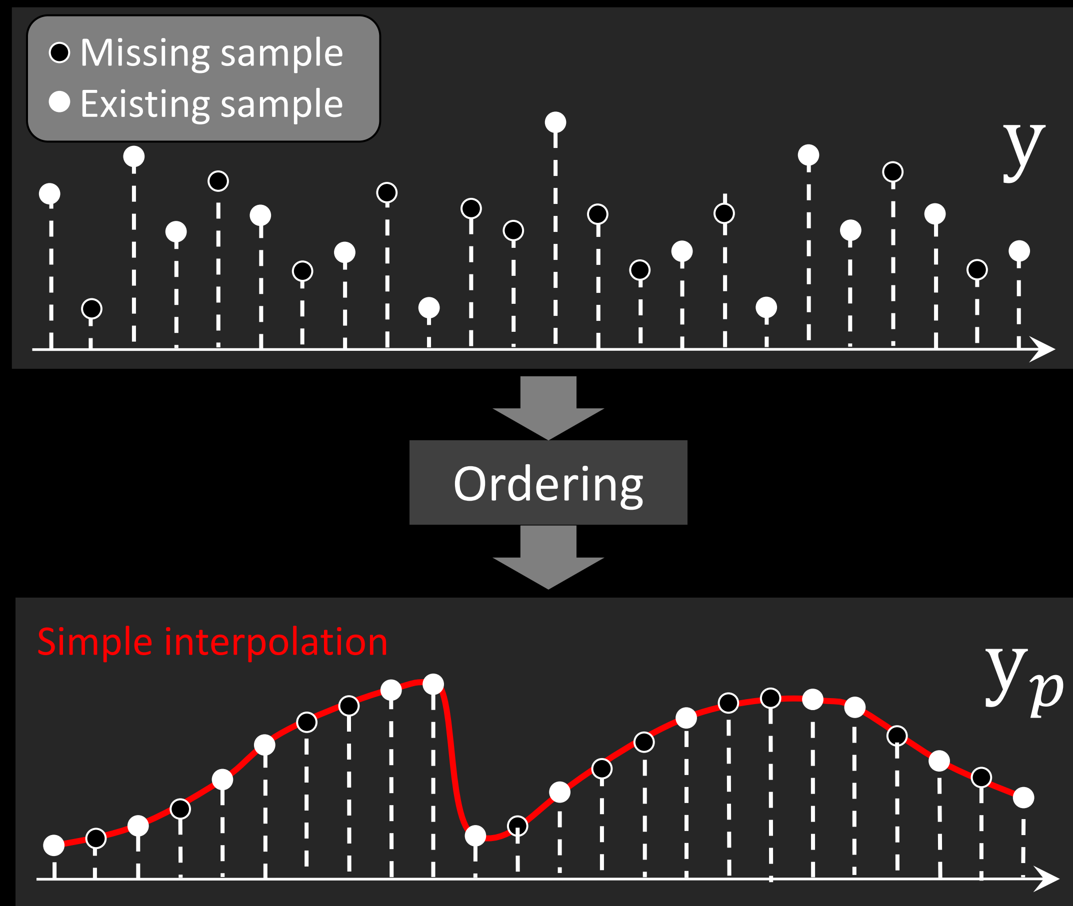
* This result is obtained with (i) cycle-spinning, (ii) sub-image averaging, and (iii) two iterations.

The Rationale

0.8 of the pixels are missing



Reconstruction: 27.15dB



Inpainting Results – Examples

Given data 80% missing pixels	Bi-Cubic interpolation	DCT and OMP recovery	1^{st} iteration of the proposed alg.	3^{rd} iteration of the proposed alg.

Inpainting Results

Reconstruction results from 80% missing pixels using various methods:

Image	Method	PSNR [dB]
Lena	Bi-Cubic	30.25
	DCT + OMP	29.97
	Proposed (1st iter.)	30.25
	Proposed (2nd iter.)	31.80
	Proposed (3rd iter.)	31.96
Barbara	Bi-Cubic	22.88
	DCT + OMP	27.15
	Proposed (1st iter.)	27.56
	Proposed (2nd iter.)	29.34
	Proposed (3rd iter.)	29.71
House	Bi-Cubic	29.21
	DCT + OMP	29.69
	Proposed (1st iter.)	29.03
	Proposed (2nd iter.)	32.10
	Proposed (3rd iter.)	32.71

Bottom line:

(1) This idea works very well;

(2) It is operating much better than the classic sparse-rep. approach; and

(3) Using more iterations always pays off, and substantially so.

Part IV – Time to Finish

Conclusions and a Bit More

Conclusions

We propose a new wavelet transform for scalar functions defined on graphs or high dimensional data clouds

The proposed transform extends the classical orthonormal and redundant wavelet transforms

We demonstrate the ability of these transforms to efficiently represent and denoise images

We also show that the obtained transform can be used as a regularizer in classical image processing Inverse-Problems

Finally, we show that using the ordering of the patches only, quite effective denoising and inpainting can be obtained

What Next ?

- Demonstrating the proposed wavelet on true data clouds/graphs
- Replace the TSP ordering by MDS?
- Exploiting the known distances?
- Sparse Representations and learned dictionaries in the ordered domain?
- Why TSP? Who says we cannot revisit patches?
- Improving the TSP approximation solver
- Replace "sub-image averaging" with a sparsifying transform
- ?
- ?
- Lifting scheme for treating clouds?

Thank you for your time

and …

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Gabriel Peyré (Paris-Dauphine, France)

Questions?