SOS Boosting for Image Deblurring Algorithms*

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***Abstract*—The work reported by Romano and Elad presents the SOS boosting – a generic recursive method for improving image denoising algorithms. Appealing properties of the SOS scheme are its flexibility to work with any denoising method, its simplicity, and its robustness. In this article we aim to generalize this to the image deblurring task. The proposed SOS procedure for deblurring is given by the following iterative steps: [S]trengthen the signal by blurring the output of the previous iteration and adding it to the blurred input image, [O]perate the deblurring algorithm on the strengthened image, and [S]ubtract the previous output from the current one. We demonstrate this procedure for several state-of-the-art methods (BM3D, EPLL and NCSR), showing the potential gain in output quality for each. As in the original SOS, we manipulate the iterative algorithm by two parameters, better controlling its steady state and rate of convergence.**

I. Introduction

Image denoising is a highly researched topic. A noisy image $\mathbf{y} \in \mathbb{R}^{r\times c}$ is given, obtained from the clean image $\mathbf{x} \in \mathbb{R}^{r\times c}$ by a corruption of the form

$$\mathbf{y} = \mathbf{x} + \mathbf{v}, \tag{1}$$

where the vector $\mathbf{v} \in \mathbb{R}^{r\times c}$ stands for an additive i.i.d and zero mean Gaussian noise. In our notation hereafter, $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{v}$ are represented as column vectors of length rc, after lexicographic ordering. A denoising algorithm $f(\cdot)$ outputs an approximation $\hat{\mathbf{x}} = f(\mathbf{y})$ of the clean image $\mathbf{x}$. Many sophisticated denoising algorithms have been proposed over the years, among those we mention K-SVD [1], EPLL [2], NCSR [3], BM3D [4] and others [5], [6], [7], [8], [9].

The work reported in [10] presented the SOS iterative boosting scheme for general image denoising algorithms. This method is described by the following three steps:

1) $Strengthen$ the signal by adding the restored result to the noisy input image.
2) $Operate$ the denoising algorithm on the strengthened image.
3) $Subtract$ the previous output from the restored strengthened image.

As shown in [10], due to the better signal-to-noise ratio (SNR) of the signal-strengthened image (compared to the original input one), an improved and more efficient denoising is achieved. The SOS boosting for image denoising is flexible, being able to work with any denoising method. The work in [10] has shown its applicability to various state-of-the-art algorithms (e.g., K-SVD [1], BM3D [4], EPLL [2], NLM [5]). Among the main appealing properties of the SOS scheme are its simplicity and robustness.

In this article we aim to generalize the SOS to the image deblurring task. Consider a blurred image obtained by a corruption of the form

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{v}, \tag{2}$$

where $\mathbf{H} \in \mathbb{R}^{rc\times rc}$ is a given blurring matrix. Our goal is the recovery of $\mathbf{x}$ from $\mathbf{y}$, assuming that $\mathbf{H}$ is known, and we denote such a process by $\hat{\mathbf{x}} = g(\mathbf{y})$.

The SOS scheme for deblurring algorithms is more challenging, as the measurement vector $\mathbf{y}$ and the recovered image $\hat{\mathbf{x}}$ are in different domains - one is blurry while the other is sharpened. We propose a process given by the same steps as in the original SOS, with a minor, yet critical, change in the first step: $Strengthen$ the signal by **blurring the output of the previous iteration** and adding it to the blurred input image. We demonstrate this procedure for several state-of-the-art methods (BM3D [11], EPLL [2] and NCSR [3]), showing the potential gain in output quality for each of these methods.

This paper is organized as follows: In section II we review the SOS boosting for image denoising algorithms and generalize it to the image deblurring task. In section III we present our experiments and their results. Conclusions and future research proposals are discussed in section IV.

II. SOS Boosting for Image Deblurring

In the following section we review the SOS boosting for denoising algorithms, adapt it for the deblurring task and discuss its generalization by two parameters that control its steady-state outcome, the requirements for convergence and its rate.

A. SOS for Image Denoising

The work done in [10] follows the idea that an emphasis of the signal over the noise could allow the denoiser to clean the image better. By leveraging this idea, [10] suggests $strengthening$ the signal of the noisy input $\mathbf{y}$ by adding the clean image $\hat{\mathbf{x}}^k$ to it, then $operating$ the denoising algorithm, $f(\cdot)$, again and finally, $subtracting$ the previous result from the current outcome. This is formulated as:

$$\hat{\mathbf{x}}^{k+1} = f(\mathbf{y} + \hat{\mathbf{x}}^k) - \hat{\mathbf{x}}^k, \tag{3}$$

where $\hat{\mathbf{x}}^0 = 0$. A key idea in the SOS boosting is the treatment of the denoising algorithm as a black-box, thus making it easy to implement on a variety of denoising methods.

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The study in [10] offers both empirical evaluation of the SOS idea, and its theoretical analysis. On the empirical side, the SOS concept is demonstrated on a series of leading denoising algorithms, (K-SVD [1], NLM [5], bilateral filter [12] and LARK [13]), showing in all these cases that improved results are within reach. On the theoretical front, this work establishes the convergence of the proposed iterative process by representing the denoising algorithm in a pseudo-linear form.

B. SOS for Deblurring

The first idea that comes to mind when trying to apply the SOS scheme on an image deblurring algorithm, $g(\cdot)$, is the following:

$$\hat{\mathbf{x}}^{k+1} = g(\mathbf{y} + \hat{\mathbf{x}}^k) - \hat{\mathbf{x}}^k, \tag{4}$$

where $\hat{\mathbf{x}}^0 = 0$. This iterative equation is the same as the one applied for the denoising case. However, note that $\mathbf{H}$, the blurring matrix, plays a critical role in the definition of the operator $g(\cdot)$. Indeed, the blurring matrix for the compound image $\mathbf{y} + \hat{\mathbf{x}}^k$ is no longer $\mathbf{H}$, but rather $\mathbf{H} + \mathbf{I}$, if we assume that $\mathbf{x}^k$ is nearly perfectly deblurred. Thus, the appropriate formulation of the SOS procedure in this case is:

$$\hat{\mathbf{x}}^{k+1} = g_{\hat{\mathbf{H}}^k}(\mathbf{y} + \hat{\mathbf{x}}^k) - \hat{\mathbf{x}}^k, \tag{5}$$

where $\hat{\mathbf{H}}^k$ is the blurring matrix referring to the image $\mathbf{y} + \hat{\mathbf{x}}^k$, which may vary from one iteration to the next, and should be evaluated somehow. This modification of $g(\cdot)$ denies us the possibility of treating the deblurring algorithm as a black-box, a major advantage of the original SOS scheme. For this reason, we turn to an alternative SOS iteration procedure, as described next.

Taking into account the desire to preserve the ability of treating the deblurring algorithms as a fixed unit, we suggest blurring the output of the previous iteration before adding it to the given image $\mathbf{y}$. This variation of SOS algorithm is described by the following steps:

1) $Strengthen$ the signal by **blurring the output of the previous iteration** and adding it to the blurred input image.
2) $Operate$ the deblurring algorithm on the strengthened image.
3) $Subtract$ the previous output from the restored strengthened image.

This small change implies that $\mathbf{H}$ is the blurring matrix of $\mathbf{y} + \mathbf{H}\hat{\mathbf{x}}^k$ (which can be also written as $\mathbf{H}(\mathbf{x} + \hat{\mathbf{x}}^k) + \mathbf{v}$). This form of the SOS algorithm can be formulated in the following way:

$$\hat{\mathbf{x}}^{k+1} = g(\mathbf{y} + \mathbf{H}\hat{\mathbf{x}}^k) - \hat{\mathbf{x}}^k, \tag{6}$$

where $\hat{\mathbf{x}}^0 = 0$, and $g(\cdot)$ remains the same 'deblurrer' throughout the iterative process. This keeps the simplicity of the SOS algorithm, thus allowing us to easily apply this scheme on various deblurring algorithms, without the need to modify them or feed them with a different blur operator.

With the above formula, our deblurring task becomes easier as we iterate, since we trade the original problem with an alternative one that has the same blur but twice weaker noise. Thus, following the same rationale of the original SOS method, better performance can be expected.

C. Parametrization of the SOS boosting

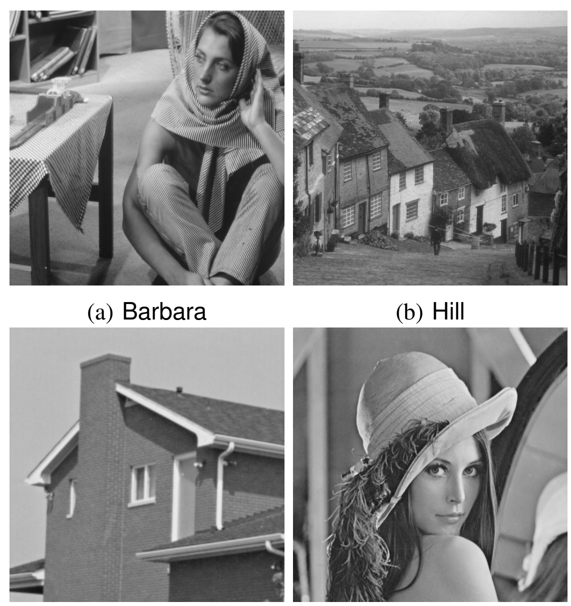
The work in [10] offered a parametric generalization of the SOS boosting formula in order to better control the steady-state outcome, and the convergence requriments and its rate. More specifically, two parameters are introduced: ρ, that controls the signal emphasis and the steady-state outcome, and τ, that changes the rate-of-convergence and softens the convergence requirements on $f(\cdot)$. Applying the same principals, we manipulate our SOS scheme in a similar way, leading to the following equation:

$$\hat{\mathbf{x}}^{k+1} = \tau g(\mathbf{y} + \rho \mathbf{H}\hat{\mathbf{x}}^k) - (\tau\rho + \tau - 1)\hat{\mathbf{x}}^k. \tag{7}$$

More on these parameters and a justification for their use is found in [10].

III. Experimental Results

We now turn to present series of experiments that demonstrate the core ability of the SOS scheme to boost image deblurring algorithms. Our focus will be put on three well performing methods - the BM3D [11], the EPLL [2], and the NCSR [3].



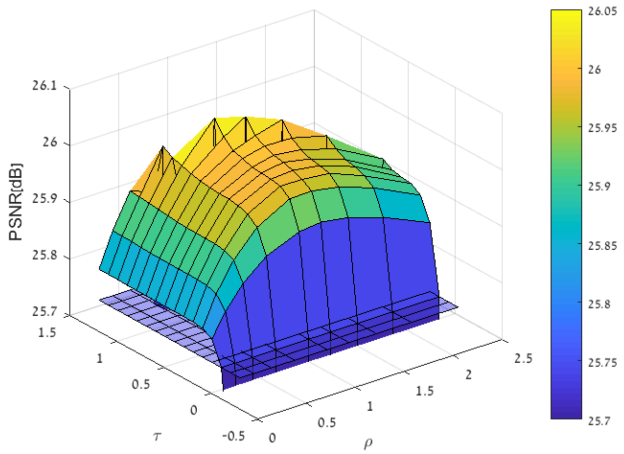
(a) Barbara (b) Hill

(c) House (d) Lena

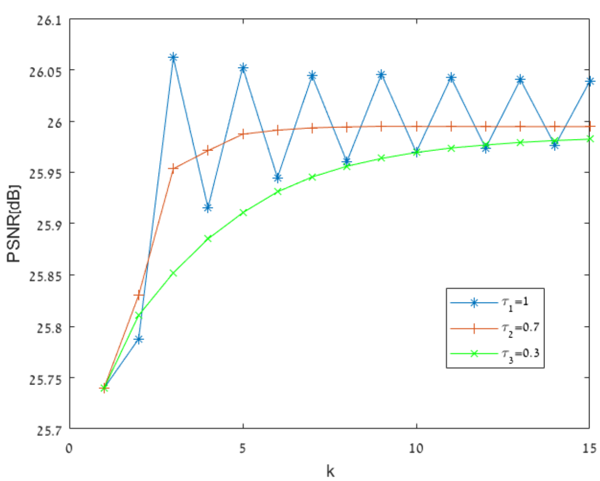
Fig. 1: The test images used in the reported experiments.

A. Experiment 1

In the first experiment our goal is to study the effects of ρ and τ on the outcome of the SOS boosting. This is done by testing these parameters with the EPLL [2] algorithm, for deblurring the image Barbara (see Figure 1a) with ($\sigma_{noise} =$



(a) PSNR of the proposed SOS boosting as a function of ρ and τ. The flat surface represents the original deblurring performance of EPLL (25.74dB). As can be seen, varying degrees of improvements are obtained for various values of ρ and τ. The best set ($\rho = 1$ and $\tau = 1$) give an increase of 0.3dB.



(b) PSNR of the proposed SOS algorithm for several choices of the parameters ρ and τ (the original PSNR of the deblurring algorithm is 25.74dB).

Fig. 2: The effect of ρ and τ on the performance and rate-of-convergence of the proposed SOS boosting. These experiments were made on EPLL [2] with a blurred version of Barbara ($\sigma_{noise} = 0.01, \sigma_{blur} = 3$).

0.01 , $\sigma_{blur} = 3$). The deblurring performance is evaluated using the PSNR, defined as $20 \log_{10}(\frac{255}{\sqrt{MSE}})$, where the MSE is the mean square error between the original image and the deblurred one. Figure 2a shows the dependence of the deblurring algorithm's performance on ρ and τ, and a visual comparison for this is given in Figure 3. As can be seen, for properly chosen values of ρ (1 in this case), the EPLL can be boosted by 0.3dB. Interestingly, the parameter τ has almost no affect on the output quality when kept in the range [0,1]. Its

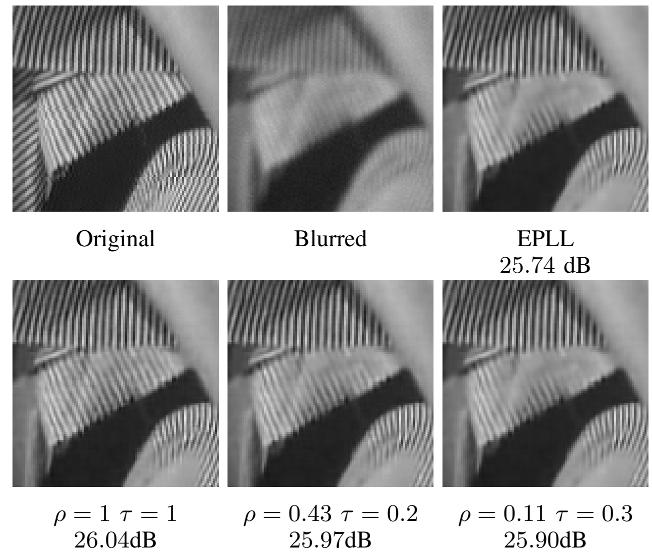


Fig. 3: Visual and PSNR comparisons between different values of the SOS parameters of a 200×200 cropped region from the blurry image Barbara ($\sigma_{noise} = 0.01, \sigma_{blurr} = 3$).

influence is on the rate-of-convergence of the SOS procedure can be seen in Figure 2b showing the PSNR per iteration for $[\tau_1, \tau_2, \tau_3] = [1, 0.7, 0.3]$ with a fixed value of $\rho = 1$. We note that in this test we set $\hat{\sigma}_{noise}$, the parameter representing the noise-level of $y + \rho \mathbf{H}\hat{\mathbf{x}}^k$, to $\sigma_{noise} = 0.01$.

Turning to the choice of $\hat{\sigma}_{noise}$ and its effect, it could be estimated automatically (e.g, using [14], [15]) or used as a fixed value chosen manually. In the all the reported experiments in this paper we chose the second option. Figure 4 shows the effect of $\hat{\sigma}_{noise}$ on the SOS boosting performance while using $\rho = 1$ and $\tau = 0.7$. As can be seen, the best performing value of $\hat{\sigma}_{noise}$ seems to be about $0.8\sigma_{noise}$, less than the true noise that exists in the degraded image. This represents a more conservative restoration step done by $g(\cdot)$, which pays off in the boosted iterative process.

To summarize, we see that a change of parameters can have a large effect on the outcome of the SOS boosting. From that, we conclude that tuning the SOS parameters for each image is important, and an improper tuning might weaken the SOS performance. Indeed, in order to demonstrate this very claim, we repeated the above test on a different image (Hill), and the optimal set of parameters for it are $\rho = 0.11$, $\tau = 0.1$, and $\hat{\sigma}_{noise} = 0.8\sigma_{noise}$.

B. *Experiment 2*

We now turn to a comprehensive experiment in which we explore the SOS boosting results on the three well-performing algorithms mentioned above (EPLL [2], BM3D [11] and NCSR [3]). Our tests now cover the performance for the images Barbara, Hill, House and Lena (see Figure 1). These images are corrupted by a 5×5 Gaussian blur kernel with a standard deviation σ_{blur}, and a zero-mean Gaussian noise with a standard deviation σ_{noise}. In this experiment we vary

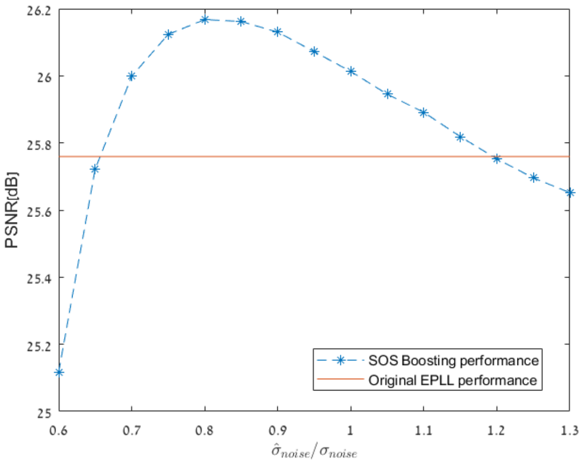


Fig. 4: PSNR of the SOS boosting as a function of $\hat{\sigma}_{noise}$. The graph is generated by operating the SOS on EPLL [2] (note that the "Original EPLL Performance" line is the performance of the EPLL algorithm with $\hat{\sigma}_{noise} = \sigma_{noise}$). The experiments were made on the blurred image Barbara ($\sigma_{noise} = 0.01, \sigma_{blur} = 3$) with $\rho = 1$ and $\tau = 0.7$.

σ_{noise} and σ_{blur} and check the SOS performance in each such setting.

In order to apply the SOS scheme, we need to set the parameters ρ, τ and a modified noise-level $\hat{\sigma}_{noise}$. In these experiments we set $\tau = 0.7$, while ρ and $\hat{\sigma}_{noise}$ are optimized per each image, algorithm and degradation, in order to understand the full potential of the SOS algorithm. Further work should be done in order to automatically tune all these parameters.

Table I presents the performance results of various deblurring algorithms and their SOS boosting outcomes. In this table we are able to observe improvements of up to 0.70dB (in terms of PSNR), though most improvements are in the range between 0.15dB and 0.45dB. This table shows that improvement is achievable on different images and various deblurring algorithms.

A visual comparison is given in Figure 5, showing the potential of the SOS boosting. Compared to the original restored results, the SOS boosting procedure offers a better restoration at sharper parts of the picture (see Barbara's pants and the windows and pipe on Hill). In addition, the SOS boosting achieves a cleaner and smoother estimation (see the drainage pipe on House).

IV. Conclusions and Future Research

The *Strengthen-Operate-Subtract* (SOS) scheme is a generic method for boosting image denoising algorithms. In this work we have generalized it for treating the image deblurring problem. Our proposed solution relies on a modification of the first step, blurring the output image before strengthening the signal, this way allowing us to treat the debluring algorithm as a black-box. Our experiments validate the effectiveness of this boosting method, when deployed with state-of-the-art algorithms. Our future work on this topic includes a theoretical analysis of the requirements for success of this method, and an estimated performance improvement in such cases.

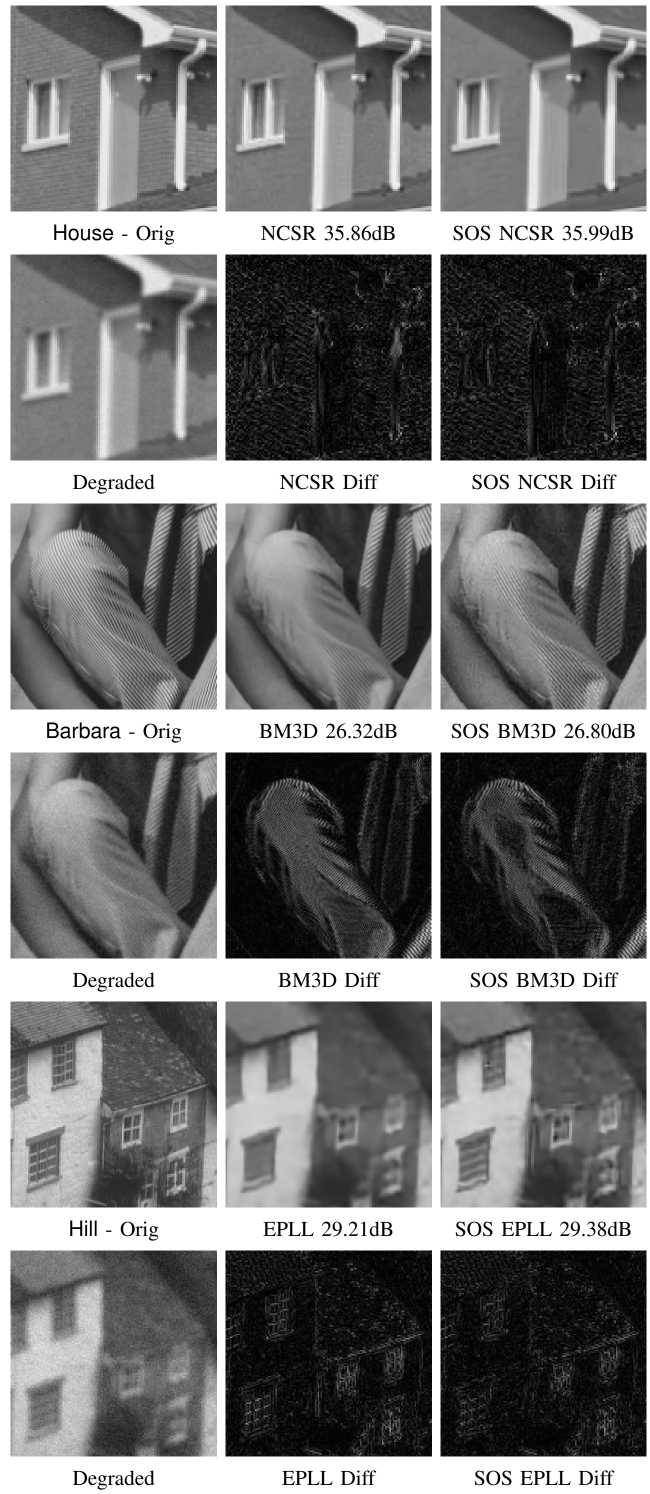


Fig. 5: Visual and PSNR comparisons between standard deblurring and SOS boosting outcomes, along with the difference between the deblurred outcomes to the original image.

TABLE I: Comparison between the deblurring results (PSNR) of EPLL [2], BM3D [11] and NCSR [3] and their SOS boosting outcomes. The best results for each test run are highlighted.

EPLL [2]													
σ_{noise}	σ_{blur}	Barbara			House			Hill			Lena		
		Orig	SOS	Imprv.	Orig	SOS	Imprv.	Orig	SOS	Imprv.	Orig	SOS	Imprv.
0.01	1	28.19	**28.89**	0.70	34.52	**33.55**	0.03	32.18	32.18	0.00	**35.49**	35.48	−0.01
0.01	3	25.75	**26.18**	0.43	**30.76**	30.73	−0.03	29.81	**29.92**	0.11	32.57	**32.58**	0.01
0.01	5	25.94	**26.20**	0.26	**30.89**	30.85	−0.04	29.89	**30.00**	0.11	32.56	**32.57**	0.01
0.03	1	25.18	**25.85**	0.67	32.99	**33.03**	0.04	30.29	**30.42**	0.13	33.36	**33.38**	0.02
0.03	3	24.16	**24.44**	0.28	**29.82**	29.80	−0.02	28.34	**28.41**	0.07	30.89	**31.00**	0.11
0.03	5	24.23	**24.48**	0.25	29.83	**29.86**	0.03	28.34	**28.47**	0.13	30.75	**30.83**	0.08
0.05	1	24.37	**24.87**	0.50	31.82	**31.87**	0.05	29.21	**29.38**	0.17	31.91	**32.07**	0.16
0.05	3	23.85	**23.99**	0.14	**29.34**	29.31	−0.03	27.66	**27.73**	0.07	29.94	**30.04**	0.10
0.05	5	23.82	**23.95**	0.13	29.22	**29.25**	0.03	27.57	**27.71**	0.14	29.82	**29.93**	0.11

BM3D [11]													
σ_{noise}	σ_{blur}	Barbara			House			Hill			Lena		
		Orig	SOS	Imprv.	Orig	SOS	Imprv.	Orig	SOS	Imprv.	Orig	SOS	Imprv.
0.01	1	30.24	**30.62**	0.38	36.02	**36.21**	0.19	33.30	**33.31**	0.01	36.01	**36.22**	0.21
0.01	3	27.32	**27.56**	0.24	34.95	**34.98**	0.03	31.75	**31.76**	0.01	34.04	**34.07**	0.03
0.01	5	27.38	**27.58**	0.20	34.93	**34.98**	0.05	31.82	**31.83**	0.01	33.96	**34.01**	0.05
0.03	1	26.32	**26.80**	0.48	33.94	33.94	0.00	30.99	**31.02**	0.03	33.74	33.74	0.00
0.03	3	24.78	**25.09**	0.31	32.51	**32.54**	0.03	29.73	**29.80**	0.07	31.94	**31.97**	0.03
0.03	5	24.91	**25.28**	0.37	32.53	**32.57**	0.04	29.68	**29.75**	0.07	31.81	**31.85**	0.04
0.05	1	25.03	**25.36**	0.33	32.63	32.63	0.00	29.75	**29.86**	0.11	32.31	**32.36**	0.05
0.05	3	24.18	**24.30**	0.12	31.22	**31.27**	0.05	28.67	**28.85**	0.18	30.73	**30.86**	0.13
0.05	5	24.24	**24.42**	0.18	31.19	**31.28**	0.10	28.64	**28.81**	0.17	30.59	**30.73**	0.14

NCSR [3]													
σ_{noise}	σ_{blur}	Barbara			House			Hill			Lena		
		Orig	SOS	Imprv.	Orig	SOS	Imprv.	Orig	SOS	Imprv.	Orig	SOS	Imprv.
0.01	1	31.26	31.26	0.00	35.86	**36.12**	0.26	33.06	**33.12**	0.06	35.82	**35.99**	0.17
0.01	3	23.89	**23.90**	0.01	**29.68**	29.61	−0.08	27.93	**27.94**	0.01	29.84	**29.86**	0.02
0.01	5	23.12	**23.13**	0.01	26.60	**26.61**	0.01	26.10	**26.14**	0.04	27.44	**27.48**	0.04
0.03	1	27.15	**27.17**	0.02	30.51	**31.12**	0.61	29.56	**29.59**	0.03	30.86	**31.00**	0.14
0.03	3	23.24	**23.26**	0.02	27.61	**27.65**	0.04	26.69	26.69	0.00	27.80	**27.81**	0.01
0.03	5	22.55	**22.57**	0.02	25.32	**25.35**	0.03	25.27	**25.29**	0.02	26.18	**26.29**	0.11
0.05	1	24.92	**25.08**	0.16	26.87	**27.49**	0.62	26.80	**26.97**	0.17	27.28	**27.85**	0.57
0.05	3	22.52	**22.78**	0.26	25.80	**26.11**	0.31	25.32	**25.76**	0.44	26.03	**26.36**	0.33
0.05	5	22.02	**22.15**	0.13	24.10	**24.33**	0.23	24.38	**24.50**	0.12	25.07	**25.37**	0.30

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