Image Denoising

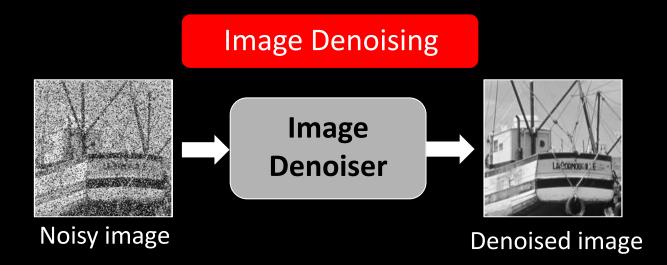
Michael Elad

Computer Science Department The Technion - Israel Institute of Technology Haifa 32000, Israel





This Lecture is About ...



Removal of noise from images is a heavily studied problem in image processing

In this talk we expand on recent discoveries and developments around this seemingly dead topic



Our Agenda

- 1. Brief Introduction & History
- 2. Image Denoising: The Classic Era
- 3. The Deep Learning Revolution
- 4. Synergy: Classic + Deep Learning

5. Our Focus Today: Denoising for ...

- Solving general inverse problems
- Image Synthesis
- High perceptual quality recovery

6. Summary



Introduction & History



So, Let's Talk About ...

Image Denoising

or more accurately

Removal of White Additive Gaussian Noise from an Image

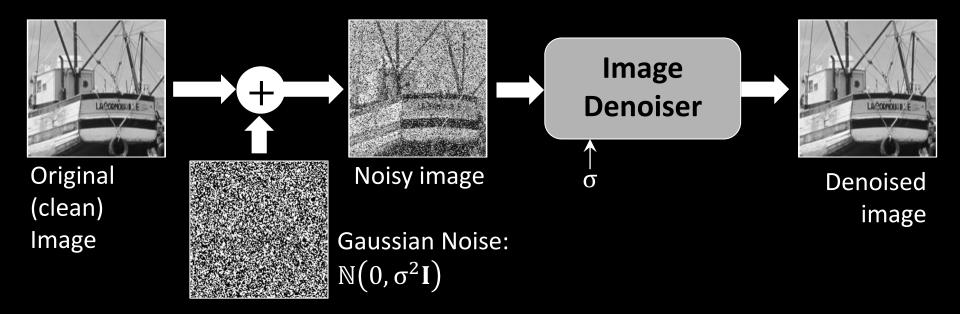
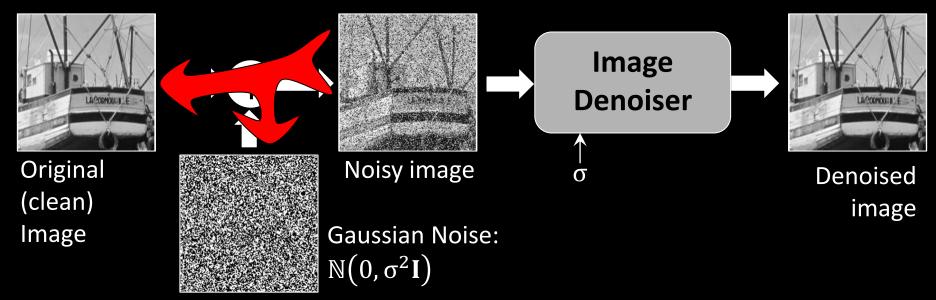




Image Denoising is Challenging

Image denoising is far from trivial task! Why?

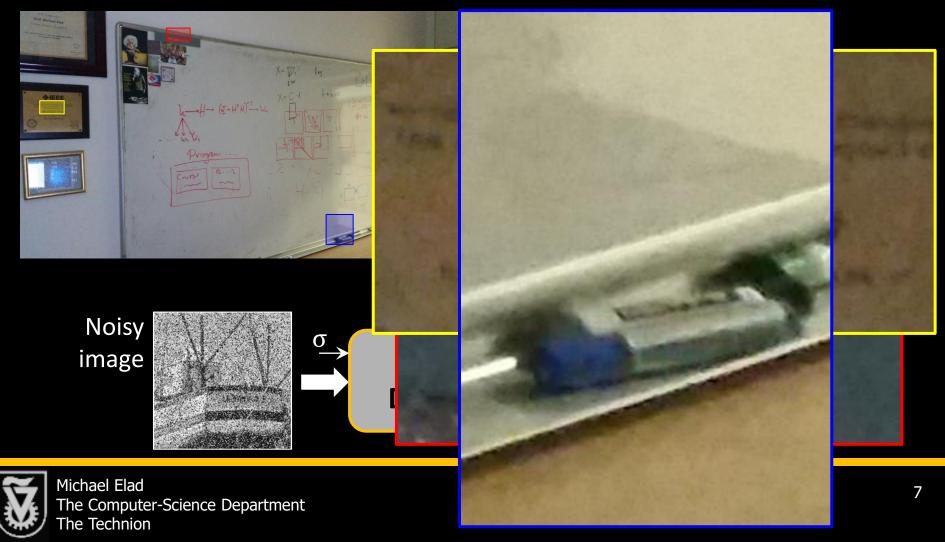
- Because our goal is to remove noise as much as possible while preserving the details in the image
- Denoising is essentially a highly ill-posed separation task





Why Work on Image Denoising?

1. Practical: It is a real-world problem, arising in all cameras,



Why Work on Image Denoising?

- 1. Practical: It is a real-world problem, arising in all cameras,
- 2. Front-Gate to Image Processing: Being the simplest inverse problem, it is a platform for assessing new ideas in our field, &
- 3. Other Uses for the Denoiser Engine: Recent work has shown that given a denoiser, there are other fascinating uses for it that go far beyond noise removal



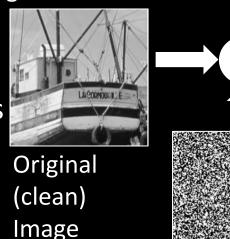


Why Assume Gaussian Noise?

□ The Gaussian case is more common and much more important

When considering a Poisson noise,

- High count of photons The distribution gets closer and closer to the Gaussian case
- Low-count Poisson-distributed image can be converted to a Gaussian-noisy one by Anscomb - Variance Stabilizing Transform
- Many of the developed ideas for the Gaussian case can be converted to other noise models
- MMSE denoisers for the
 Gaussian case are of extreme
 theoretical value (see later)



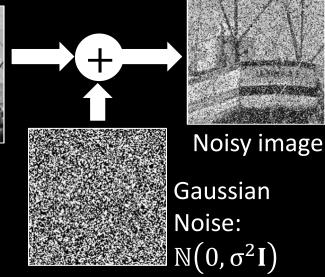
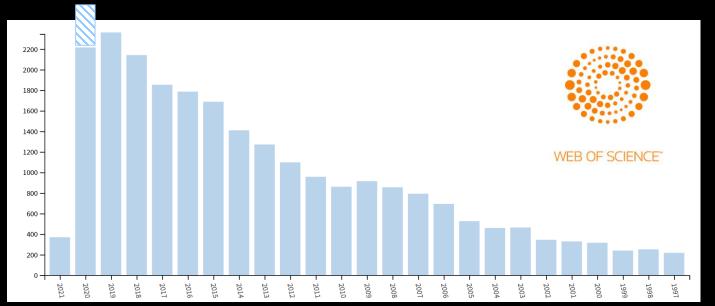




Image Denoising: Little bit of History

Roughly speaking, there are ~25,000 papers* on this subject, offering algorithms, theoretical analysis and so much more



My speculation

* Search done on June 1st in WoS, topic: ((image or video) and (denoising or (noise and remov) or clean))



Image Denoising: Little bit of History

Citing Arti	cles:
USA: 1	40524
China:	45284
Germany:	29272
France:	35585
England:	24090
Canada:	18325
Spain:	17880
Israel:	13988
Australia:	13358
Switz.:	12504
Japan:	12389
Italy:	11754
Netherland:	10455
India:	8830
Finland:	7842
Korea:	7558
Belgium:	5027
Singapore:	4964
Brazil:	4849
Taiwan:	4134
Iran:	3112
Russia:	2595

This research comes from all over the globe 6.805 1,534 542 429 393 386 903 **PEOPLES R CHINA** IRAN GERMANY CANADA AUSTRALIA NETHERLAN SWITZERL 1,417 856 FRANCE 358 295 290 5,678 SINGAPORE BRAZIL ISRAEL 1,147 738 340 JAPAN ITALY 256 260 FINLAND RUSSIA 337 1.664 BELGIUM 1.119 624 INDIA 253 241 SPAIN 307 POLAND AUSTRIA

... and it is heavily cited



The Classic Era



Design of Image Denoising Algorithms

How can we design a denoiser?

The classic Bayesian approach (1960-2014):

- Model image content with a prior expression (e.g., forcing smoothness, sparsity, low-rank, self-similarity, ...), and
- Formulate the denoising task as an optimization problem

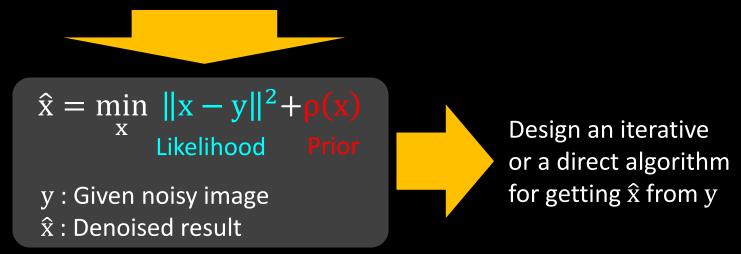
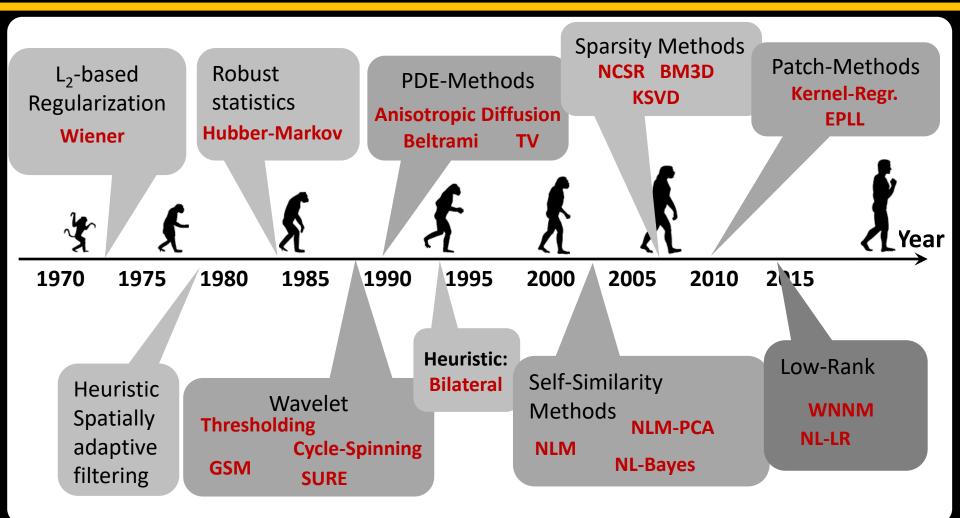




Image Denoising: Evolution





End of an Era?

This evolution of algorithms and the tendency of different methods to perform very similarly has led to a feeling that "Denoising is Dead"

IEEE TRANSACTIONS ON IMAGE PROCESSING, VOL. 19, NO. 4, APRIL 2010

895

CVPR 2011

Is D

Priyam Chatterjee, Student M

Abstract-Image denoising has been a well studied the field of image processing. Yet researchers continue tention on it to better the current state-of-the-art. Re posed methods take different approaches to the probl their denoising performances are comparable. A pertion then to ask is whether there is a theoretical limit performance and, more importantly, are we there yet? manufacturers continue to pack increasing numbers of unit area, an increase in noise sensitivity manifests itself of a noisier image. We study the performance bounds for denoising problem. Our work in this paper estimates a l on the mean squared error of the denoised result and co performance of current state-of-the-art denoising me this bound. We show that despite the phenomenal rece in the quality of denoising algorithms, some room for ment still remains for a wide class of general images, an signal-to-noise levels. Therefore, image denoising is not

Index Terms—Bayesian Cramér–Rao lower boun bias, bootstrapping, image denoising, mean squared en

I. INTRODUCTION

MAGE denoising has been a well-studied the image processing community and continue



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Natural Image Denoising: Optimality and Inherent Bounds

Anat Levin and Boaz Nadler Department of Computer Science and Applied Math The Weizmann Institute of Science

Abstract

The goal of natural image denoising is to estimate a clean version of a given noisy image, utilizing prior knowledge on the statistics of natural images. The problem has been studied intensively with considerable progress made in recent years. However, it seems that image denoising algorithms are starting to converge and recent algorithms improve over previous ones by only fractional dB values. It is thus important to understand how much more can we still improve natural image denoising algorithms and what are the inherent limits imposed by the actual statistics of the data. The challenge in evaluating such limits is that constructing proper models of natural image statistics is a long standing and yet unsolved problem.

To overcome the absence of accurate image priors, this paper takes a non parametric approach and represents the distribution of natural images using a huge set of 10^{10} patches. We then derive a simple statistical measure which provides a lower bound on the optimal Bayesian minimum mean square error (MMSE). This imposes a limit on the best possible results of denoising algorithms which utilize a ever, it seems that the performance of denoising algorithms is starting to converge. Recent techniques typically improve over previous ones by only fractional dB values. In some cases the difference between the results of competing algorithms is so small and inconclusive, that one actually has to successively toggle between images on a monitor to visually compare their denoising quality. This raises the question of whether the error rates of current denoising algorithms can be reduced much further, or whether there are inherent limitations imposed by the statistical structure of natural images? The goal of this paper is to derive a lower bound on the best possible denoising error under a well defined statistical framework. Such a bound can help us understand if there is hope to significantly improve the current stateof-the-art image denoising with even better algorithms, or whether we have nearly approached the fundamental limits.

Understanding the limits of natural image denoising is also important as an instance of a more fundamental computer and human vision challenge: modeling the statistics of natural images and understanding the inherent limits of their statistical power. Several works attempted to estimate the entropy of natural images [15, 4]. However, there is And so, somewhere around 2010-2012, the general feeling in our community was that ...

We are currently touching the ceiling in denoising performance and chances of improving them are very slim

There is no point in devising new denoising methods

Work in this field has diminishing returns

W⁴: Well, We Were Wrong !



End of an Era?

Wrong ? How?

The past decade has taught us that image denoising is still

very much alive and kicking

due to several branches of novel activity on:

- Obtaining better performing denoisers with deep learning
- New frontiers in denoising:
 - Better adaptation to image content
 - Denoising strategies that go beyond PSNR
 - Identifying alternative methods for designing/training denoisers
 - Extending the denoising task to realistic noise, and
- Discovering new ways for leveraging denoisers for other needs



The Deep Learning Revolution



Design of Algorithms: Take 2

How can we **ALTERNATIVELY** design a denoiser?

The machine learning approach (2012-Now):

- Gather a LARGE dataset of clean images $\{x_k\}_{k=1}^N$
- Add AWGN these images: $\{y_k = x_k + n_k\}_{k=1}^N$
- Define a parametric denoising machine $D_{\theta}(y)$
- Train $D_{\theta}(\bullet)$ by setting its parameters θ :



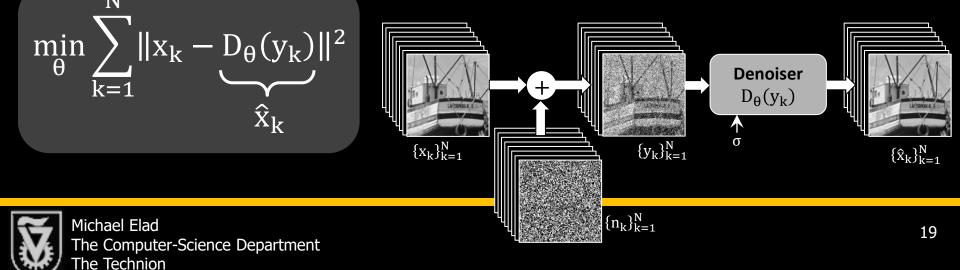


Image Denoising: A Paradigm Shift

How can we design a denoiser?

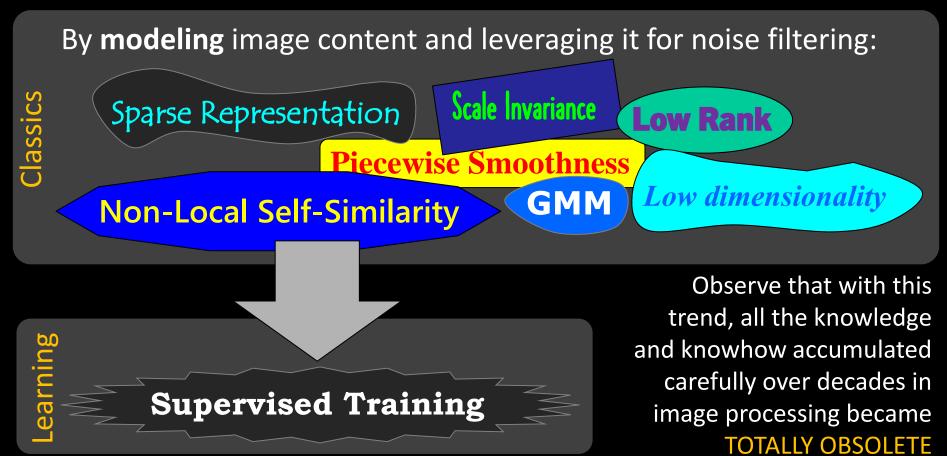
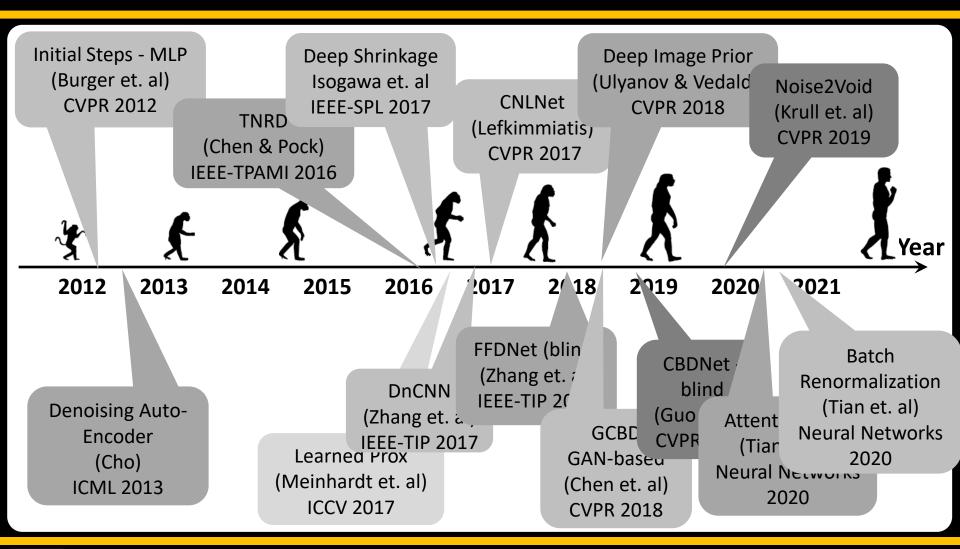




Image Denoising: Recent Evolution





Synergy: Classics + Deep Learning



Image Denoising: Return of the Classics

- In recent years deep learning is ruling the image denoising domain, pushing aside all the classical methods, along with their great achievements
- Recently, however, we do see a synergy between the two paradigms
- Recall: In building a supervised deep learning denoiser solution, we operate along the following lines:



Define an architecture for the Denoiser



Define a cost function (loss) to optimize



Train and hope for good generalization



Image Denoising Architectures

So, how do we choose an architecture for a given task?

Option 1 - Copy an existing network that has shown good results in earlier work (VGG, U-Net, ...), and slightly modify it

Option 2 – Pile and Guess a series of steps that mix known pieces such as convolutions, fully connected layer, batch-norm, ReLU,

pooling, stride, skips, upscale/downscale, connections, ... and add new "tricks"

Option 3 – Neural Architecture Search

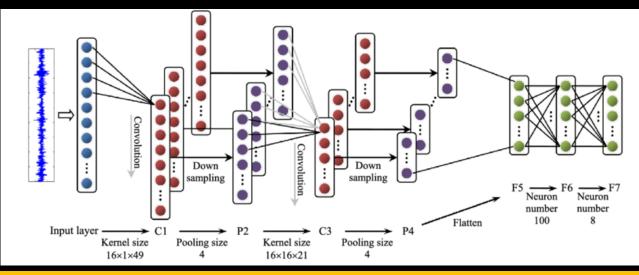
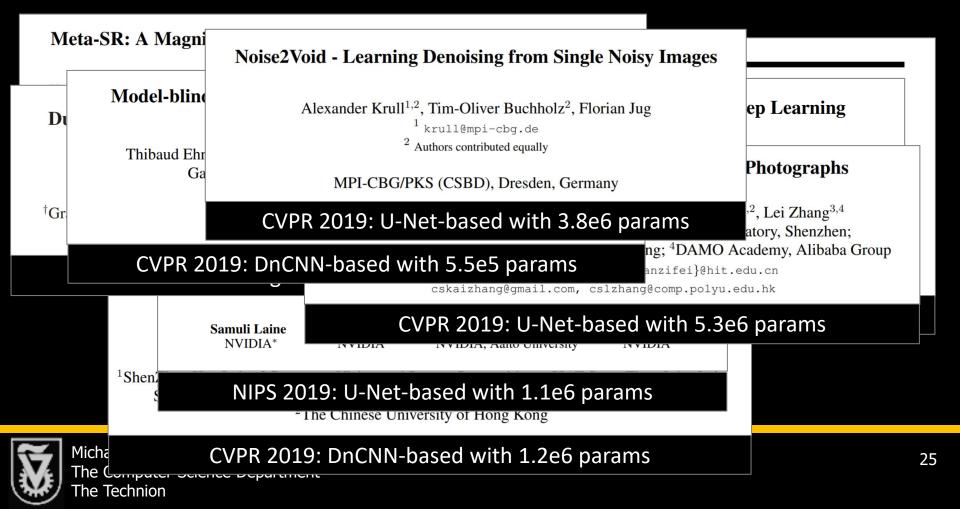




Image Denoising Architectures

Here are several paper examples from CVPR/NIPS 2019 that illustrate these architectures



Alternative Architecture Design

- Message: Do far better in choosing architectures by relying on unfolding algorithms from the classics of image processing
- □ The benefits in such architectures:
 - They are far more concise yet just as effective as leading methods
 - They are easier to train because they are lighter
 - They have the potential to break current performance barriers
 - They may bring better understanding and explainability
 - They enable better adaptation to out of distribution images
- □ Here are few representative examples:
 - Rethinking the CSC Model [Simon & Elad, NIPS `19]
 - Non-Local & Multi-Scale Denoising [Vaksman, Milanfar & Elad, CVPR (NTIRE) '20]
 - Deep KSVD Denoising [Scetbon, Milanfar & Elad, IEEE-TIP `21]
 - PatchCraft: Non-Local Video Denoising [Vaksman, Elad & Milanfar, ArxiV `21]



Our Focus Today: Recent Discoveries



Our Focus Today

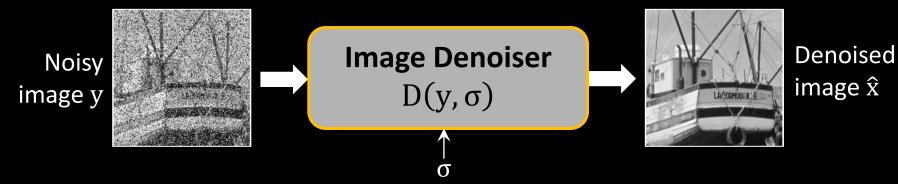
Recent findings on using denoisers for other tasks:

Discovery 1: Solving general inverse problems [2013-]

Discovery 2: Image Synthesis [2019-]

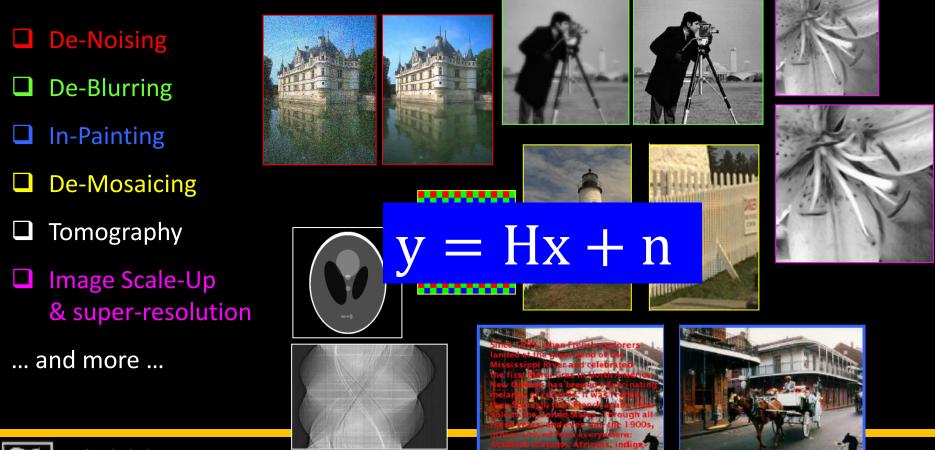
Discovery 3: High perceptual quality recovery [2021-]

We turn to describe these results





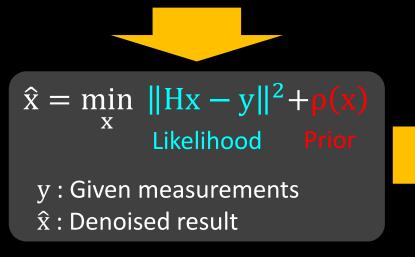
Inverse Problems: Recovery of images from corrupted measurements



How can we solve inverse problems?

We can return to the classic Bayesian approach:

- Model image content with a prior expression (e.g., forcing smoothness, sparsity, low-rank, self-similarity, ...), and
- Formulate the inversion task as an optimization problem



- This is known as MAP estimation
- It is an extension of the classic path for denoising, tailoring methods for inverse problems
- This approach leads to iterative algorithm for getting x from y
- Is there a supervised learning alternative? Definitely!



Question: Given a denoiser $D(y, \sigma)$ how can one solve inverse problems with it?

Plug-and-play priors for model based reconstruction SV Venkatakrishnan, CA Bouman, B Wohlberg 2013 IEEE Global Conference on Signal and Information Processing, 945-948	382	2013
The little engine that could: Regularization by denoising (RED) Y Romano, M Elad, P Milanfar SIAM Journal on Imaging Sciences 10 (4), 1804-1844	261	2017
Answer: Use $D(y, \sigma)$ as a regularizer		
Practical Implication: Iterated use of $D(\cdot, \sigma)$		
Simple $D(\cdot, \sigma)$ Simple $D(\cdot, \sigma)$ $D(\cdot, \sigma)$		Simple Operation



Here is (roughly) the PnP Perspective in a nutshell:

Recall: Inverse problems can be formulated as optimization tasks:

$$\hat{\mathbf{x}} = \min_{\mathbf{x}} \frac{1}{2} \|\mathbf{H}\mathbf{x} - \mathbf{y}\|^2 + \rho(\mathbf{x})$$

Let's do something "stupid" and split the unknown:

$$\hat{\mathbf{x}} = \min_{\mathbf{x}, \mathbf{v}} \frac{1}{2} \| \mathbf{H}\mathbf{x} - \mathbf{y} \|^2 + \rho(\mathbf{v}) \text{ s.t. } \mathbf{x} = \mathbf{v}$$

Now, turn the constraint into a penalty*

$$\hat{\mathbf{x}} = \min_{\mathbf{x}, \mathbf{v}} \frac{1}{2} \| \mathbf{H}\mathbf{x} - \mathbf{y} \|^2 + \rho(\mathbf{v}) + \beta \| \mathbf{x} - \mathbf{v} \|^2$$

- And solve by alternating between x and v
 - Least-Squares: $\hat{x} = \min_{x} \frac{1}{2} ||Hx y||^2 + \beta ||x v||^2$
 - A denoiser: $\hat{\mathbf{v}} = \min_{\mathbf{v}} \rho(\mathbf{v}) + \beta \|\mathbf{x} \mathbf{v}\|^2$

... and this way we got an iterated algorithm that keeps calling to a denoiser, for solving the inverse problem



^{*} The PnP uses the Augmented Lagrange which is more accurate and less sensitive to the choice of β

Here is the RED Perspective in a nutshell:

Let's start again with the formulated optimization task, and suggest a very specific regularization term: $\hat{x} = \min_{x} \frac{1}{2} ||Hx - y||^{2} + \rho(x) = \min_{x} \frac{1}{2} ||Hx - y||^{2} + \lambda x^{T} [x - D(x, \sigma)]$ Let's use the Steepest Descent $Under mild conditions^{*} the gradient of this is [x - D(x, \sigma)]$ $\hat{x}_{k+1} = \hat{x}_{k} - \mu \left[H^{T} (H\hat{x}_{k} - y) + \lambda [\hat{x}_{k} - D(\hat{x}_{k}, \sigma)] \right]$

... and this way we got an iterated algorithm that keeps calling to a denoiser, and is guaranteed to achieve the minimum

* Differentiability, local homogeneity, passivity and symmetric Jacobian (MMSE)



Here are some results for Deblurring and Super-Resolution



(a) Ground Truth



(b) Input 20.8



(a) Bicubic 20.68dB



(b) NCSR 26.79dB



(d) NCSR 28.39dB









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(c) P³-TNRD 26.61dB

PnP and RED are heavily cited and extensively studied, owing to their generality and elegance

Plug-and-play priors for model based reconstruction SV Venkatakrishnan, CA Bouman, B Wohlberg 2013 IEEE Global Conference on Signal and Information Processing, 945-948	382	2013
The little engine that could: Regularization by denoising (RED) Y Romano, M Elad, P Milanfar SIAM Journal on Imaging Sciences 10 (4), 1804-1844	261	2017

Follow-up work focuses on

- Proving convergence to the desired solution and tying these to properties of the permissible denoisers (e.g. MMSE ...)
- Deployment in various applications
- Creation of new variants of these two methods ... and ...

PnP/RED can be used to define well-motivated architectures for solving general inverse problems, built around a core learned denoising engine



Discovery 2: Image Synthesis

- In recent years, and with the deep-learning revolution, there is a growing interesting is synthesizing images "out of thin air"
- The popular tool of interest is called GAN Generative Adversarial Network, built of two competing networks – a generator and a critique
- □ Why synthesize? Because
 - We can, and it is fascinating
 - It testifies that we have seized the distribution of images, and
 - It could be used for other needs
 - Could we synthesize images differently?



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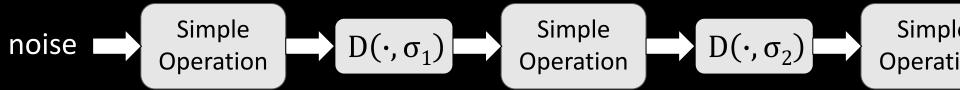


Question: Given a denoiser $D(y, \sigma)$ how can one synthesize images with it?

Generative modeling by estimating gradients of the data distribution Y Song, S Ermon arXiv preprint arXiv:1907.05600	104	2019
Improved techniques for training score-based generative models Y Song, S Ermon arXiv preprint arXiv:2006.09011	20	2020
Solving linear inverse problems using the prior implicit in a denoiser Z Kadkhodaie, EP Simoncelli arXiv preprint arXiv:2007.13640	4	2020

Answer: Use $D(y, \sigma)$ as a **Projector** onto the image manifold

Practical Implication: Iterated use of $D(\cdot, \sigma)$ with varying σ





Here is the core idea in a nutshell:

Our goal: draw a sample from the distribution of images P(x)

- Start with a random noise image \hat{x}_0
- Climb to a more probable image by the iterative equation:

 $\hat{x}_{k+1} = \hat{x}_k + a \cdot \frac{\nabla \log P(\hat{x}_k)}{\nabla \log P(\hat{x}_k)} + b \cdot z_k$ (Langevin Dynamics)

 $\begin{array}{l} \mbox{This is known as the Score} \\ \mbox{Function and it is approximately} \\ \mbox{proportional to } [\widehat{x}_k - D(\widehat{x}_k,\sigma)] \\ \mbox{for a small value of } \sigma \end{array}$

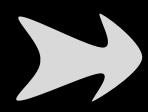
This suggests an implicit relation between MMSE denoisers and Priors: $D(x, \sigma) \leftrightarrow P(x)$

... and this way we got an iterated algorithm that keeps calling to a denoiser, and is guaranteed to obtain a sample from P(x)



In practice, instead of the plain Langevin with a fixed (and small) value of σ we use the Annealed Langevin Algorithm Blurred Image that considers a sequence of blurred priors:

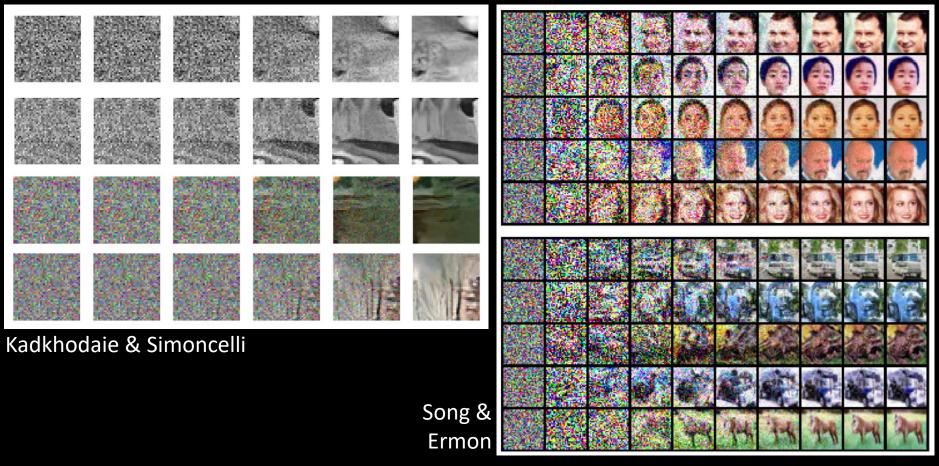
$$\begin{split} P(\mathbf{x} + \mathbf{v}) & \text{for } \mathbf{v} \sim \mathbb{N} \Big(\mathbf{0}, \sigma_k^2 \mathbf{I} \Big) \\ &= P(\mathbf{x}) \otimes \mathbf{c} \cdot \exp \left\{ -\frac{1}{2\sigma^2} \|\mathbf{x}\|^2 \right\} \\ & \text{with } \sigma_0 > \sigma_1 > \sigma_2 \quad \cdots > \sigma_N > 0 \end{split}$$



The core idea: start by drawing from a wider distribution and gradually narrow it, leading to faster and sampling performance



Does it work? Here are some results





Claim: diffusion-based methods are the best in image synthesis









BigGAN (FID 6.95)



Diffusion (FID 4.59)

Diffusion Models Beat GANs on Image Synthesis

Prafulla Dhariwal* OpenAI prafulla@openai.com Alex Nichol* OpenAI alex@openai.com

Abstract

We show that diffusion models can achieve image sample quality superior to the current state-of-the-art generative models. We achieve this on unconditional image synthesis by finding a better architecture through a series of ablations. For conditional image synthesis, we further improve sample quality with classifier guidance: a simple, compute-efficient method for trading off diversity for fidelity using gradients from a classifier. We achieve an FID of 2.97 on ImageNet 128×128, 4.59 on ImageNet 256×256, and 7.72 on ImageNet 512×512, and we match BigGAN-deep even with as few as 25 forward passes per sample, all while maintaining better coverage of the distribution. Finally, we find that classifier guidance combines well with upsampling diffusion models, further improving FID to 3.94 on ImageNet 256×256 and 3.85 on ImageNet 512×512 . We release our code at https://github.com/openai/guided-diffusion.

1 Introduction

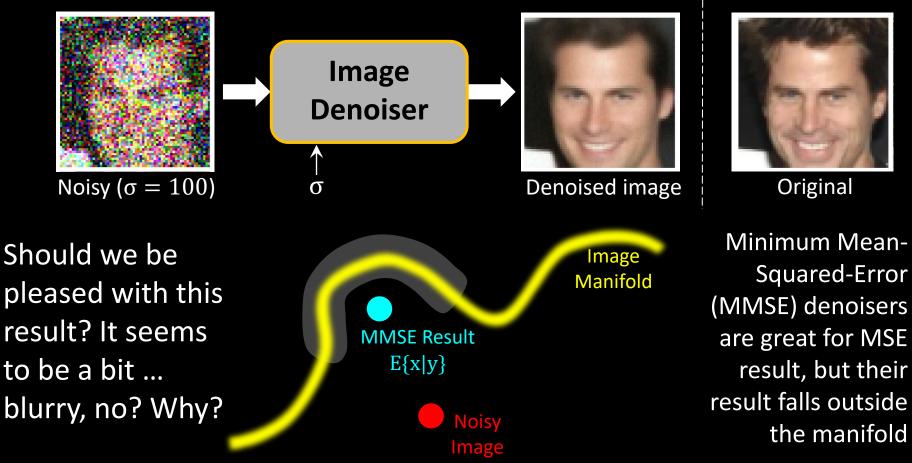


Figure 1: Selected samples from our best ImageNet 512×512 model (FID 3.85)



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Suppose that we need to denoise the following image:





Question: How can we denoise an image while targeting "High Perceptual Quality"?

High Perceptual Quality Image Denoising with a Posterior Sampling CGAN G Ohayon, T Adrai, G Vaksman, M Elad, P Milanfar arXiv preprint arXiv:2103.04192	2021
Stochastic Image Denoising by Sampling from the Posterior Distribution B Kawar, G Vaksman, M Elad arXiv preprint arXiv:2101.09552	2021
Answer: Denoise by sampling from the posterior $P(x y)$	
Answer: Denoise by sampling from the posterior $P(x y)$	

- Training a deep denoiser via CGAN, or
- Iterated use of an MMSE denoiser $D(\cdot, \sigma)$

These methods produce a multitude of possible solutions



Let's have a closer look at the Stochastic Image Denoiser:

Task: Draw a sample from P(x|y) where $[y = x + n, n \sim \mathbb{N}(0, \sigma_0^2 \mathbf{I})]$

- Start with a random noise image \hat{x}_0
- Climb to a more probable image by the iterative equation:

$$\hat{x}_{k+1} = \hat{x}_k + a \cdot \nabla \log P(\hat{x}_k | y) + b \cdot z_k \qquad \text{Langevin with a conditional Score}$$

$$= \nabla \log P(\hat{x}_k) + \nabla \log P(y | \hat{x}_k)$$
Bayes rule
$$= \hat{x}_k - D(\hat{x}_k, \sigma) + \nabla \log P(y | \hat{x}_k)$$
Approx Score A Gaussian Distribution

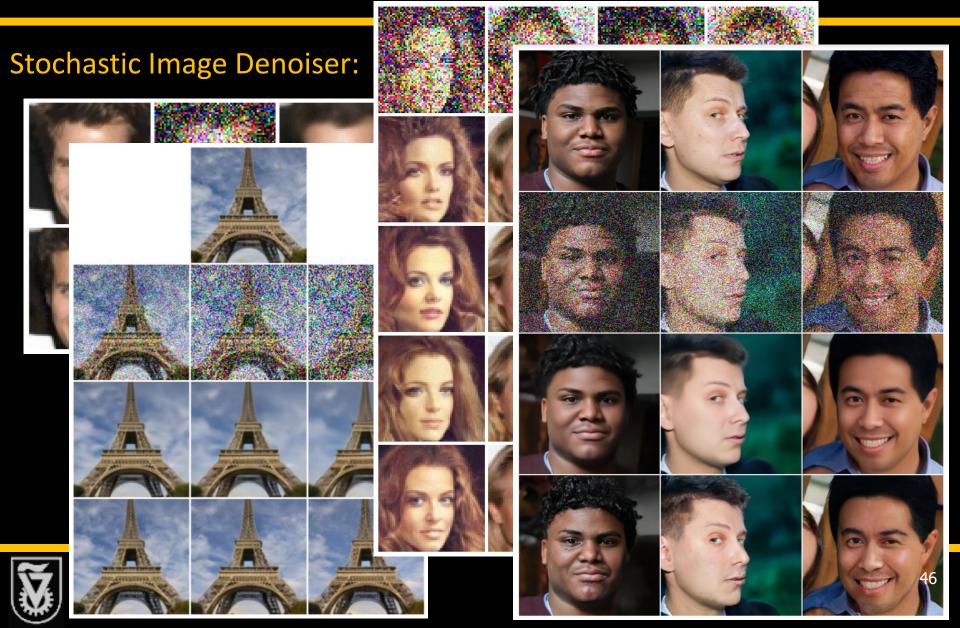


Let's have a closer look at the Stochastic Image Denoiser:

$$\nabla \log P(\hat{x}_k|y) = \hat{x}_k - D(\hat{x}_k, \sigma) + \nabla \log P(y|\hat{x}_k)$$

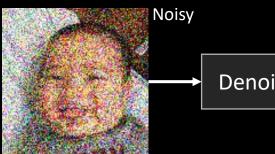
- As we use the Annealed Langevin algorithm, there are two noise signals to consider:
 - Measurement's noise: $n \sim \mathbb{N}(0, \sigma_0^2 \mathbf{I})$
 - \circ Synthetic annealing noise: $v \sim \mathbb{N}(0, \sigma_k^2 \mathbf{I})$ for $\sigma_0 > \sigma_1 > \sigma_2 \quad \cdots \quad > \sigma_N > 0$
- Implication: We recover
 a sequence of gradually less
 noisy images \$\hat{x}_k\$ where their $x_k D(\hat{x}_k, \sigma_k) + \frac{y \hat{x}_k}{\sigma_0^2 \sigma_k^2}$ noise v is assumed to be a portion of n

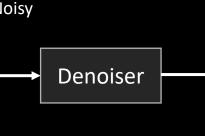




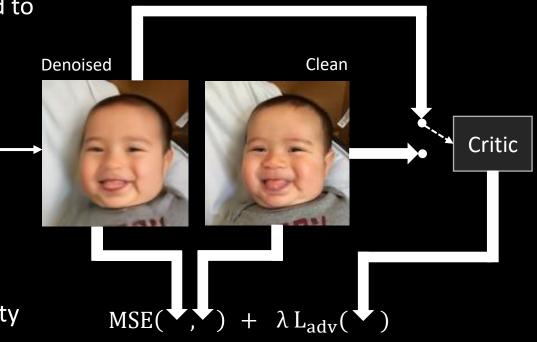
Let's have a closer look at the Conditional GAN Denoiser:

- Typical design approach: Optimize a distortion measure (e.g. MSE) between the denoised and the ideal images
- Adversarial loss could be added to improve the perceptual quality



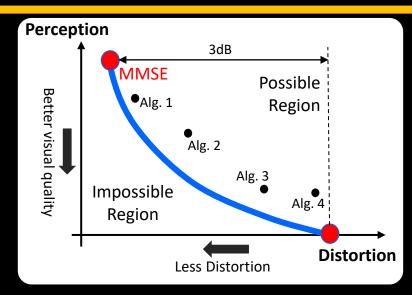


However, when used together, we get a compromise between distortion and perceptual quality





- □ For ill-posed restoration tasks, perceptual quality performance comes at the expense of its distortion [Blau & Michaeli 2017]
- We aim for best perceptual quality
- The posterior distribution attains perfect perceptual quality, compromising 3dB compared to the MMSE [Blau & Michaeli 2017]
- We propose to sample from the posterior via a Conditional GAN mechanism (PSCGAN)



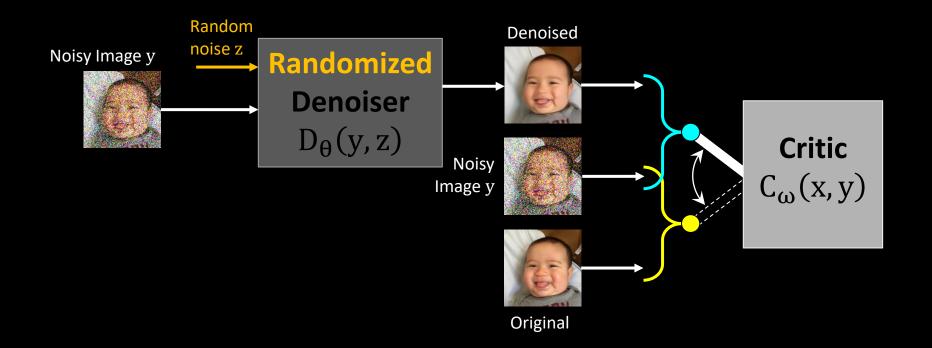
Samples from $P_{X|Y=y}$

$$x \sim P_X$$
 $y \sim P_{Y|X=x}$





The PSCGAN Architecture:





What about the Loss?

CGAN optimization leads to posterior sampling [Adler et al. 2018]:

 $\overline{\min_{\theta} \max_{\omega} \mathbb{E}_{X,Y}} \left[C_{\omega}(x,y) \right] - \mathbb{E}_{D_{\theta},Y,Z} \left[C_{\omega}(D_{\theta},y) \right]$

However, this requires an unavailable balanced dataset to train on (many x's for each y and many y's for each x)

On the other hand, we would like to avoid a penalty of the form

 $\mathbb{E}_{X,Y,Z}\big[\|x-D_{\theta}(y,z)\|_2^2\big]$

Our remedy: adding an MMSE oriented penalty term:

$\mathbb{E}_{X,Y}\big[\|x - \mathbb{E}_{z}[D_{\theta}|y]\|_{2}^{2}\big]$

This gives the MMSE result "for free" (averaging many instances)



CGAN:



Time to Summarize



Summary

Image Denoising ... Not What You Think

- 1. There are so many opportunities for better understanding, designing, and proposing creative usage of image denoisers
- 2. Despite the fact that this has not been a talk about Deep-Learning, the presence of this field in image denoising is prominent

